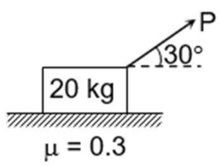
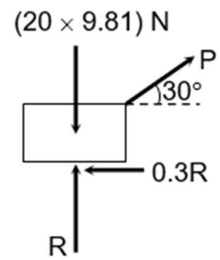
(Affiliated to University of Mumbai)	***End Semester Examination (R-24) SH 2025*** ***Answer Key with marking scheme***
Branch: All	Course: Applied Mechanics and Robot Dynamics
Year/ Semester: FY B.Tech / I	Course code: FEC105
Time: 02 hours	Marks: 60

		Marks
Q. 1	Attempt any **THREE.** (All questions carry equal marks)	**15**
A.	Find the force P required to just move the 20 kg block to the right.	05

Take $\mu_s = 0.3$





FBD of block

Consider the body having impending slipping towards the right. Hence the frictional force on the block will act towards the left and will have a magnitude given by 'μ_s R'

$\Sigma F_x = 0$, $\quad -0.3\,R + P\cos30^\circ = 0$

$\Sigma F_Y = 0$, $\quad R + P\sin30^\circ - 20(9.81) = 0$

on solving these two equations simultaneously, we get

R = 167.23 N, P = 57.93 N

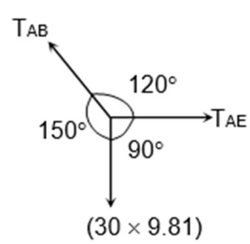
B.	A 30 kg pipe is supported at A by a system of five chords. Determine the force in each chord for equilibrium.	05

For forces acting at junction A, using Lami's theorem,

$$\frac{30 \times 9.81}{\sin120^\circ} = \frac{T_{AE}}{\sin150^\circ} = \frac{T_{AB}}{\sin90^\circ}$$

$\therefore$ $T_{AE} = 169.9$ N

$T_{AB} = 339.8$ N

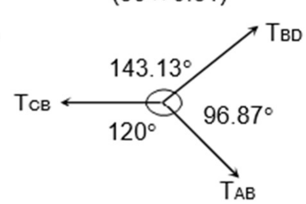


For forces acting at junction B, using Lami's theorem,

$$\frac{339.8}{\sin143.13^\circ} = \frac{T_{CB}}{\sin96.87^\circ} = \frac{T_{BD}}{\sin120^\circ}$$

$\therefore$ $T_{CB} = 562.3$ N

$T_{BD} = 490.5$ N



C.	Two concurrent forces P and Q acts at O such that their resultant acts along x-axis. Determine the magnitude of Q and hence the resultant.	05

(A) As resultant is given along x axis.

$R_x = R$ and $R_y = 0$

$R_y = \Sigma F_y$ ($\uparrow$ +ve)

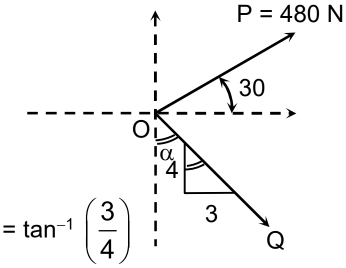
$\therefore\ 0 = 480 \sin 30 - Q \cos \alpha$

$Q = 300$ N

and $R_x = \Sigma F_x$ ($\rightarrow$ +ve)

$\therefore\ R = 480 \cos 30 + Q \sin \alpha = 595.69$ N

$R = 595.69$ N ($\rightarrow$)



$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$

$= 36.87°$

D. A particle starts from rest and moves along a straight line. The velocity of the particle varies as per the equation $v = -t^2 + 20t$ where v and t are in m/s and seconds respectively. Find the position and acceleration of the particle at t = 5 sec, 10 sec. — **05**

$$v = -t^2 + 20t \quad \ldots (1)$$

$$\therefore\ a = \frac{dv}{dt} = -2t + 20 \quad \ldots (2)$$

To obtain x from v we need to integrate

From (1),

$$v = \frac{dx}{dt} = -t^2 + 20t$$

$$\therefore \int dx = \int(-t^2 + 20t)\,dt$$

$$\therefore\ x = \frac{-t^3}{3} + 10t^2 + c$$

Taking the starting point of the particle as the origin, we have, at t = 0, x = 0

$\therefore\ 0 = 0 + 0 + c$

$\therefore\ c = 0$

$$\therefore\ x = -\frac{t^3}{3} + 10t^2 \quad \ldots (3)$$

At t = 5 sec,

$x_5 = 208.33$ m , $a_5 = 10$ m/s²

At t = 10 sec,

$x_{10} = 666.67$ m, $a_{10} = 0$

Q.2 Attempt any **THREE.** (All questions carry equal marks) — **30**

A. I) Find the resultant of coplanar force system given below and locate the same on AB with due consideration to the applied moment. — 06

Let us first find out all the angles,

From the figure,

$\alpha = \tan^{-1}(120/160) = 36.87°$

and $\beta = \tan^{-1}(160/120) = 53.13°$

$\sum F_X = 400\cos\beta - 320 - 200\cos\alpha + 50\cos\alpha$

$= -200$ N

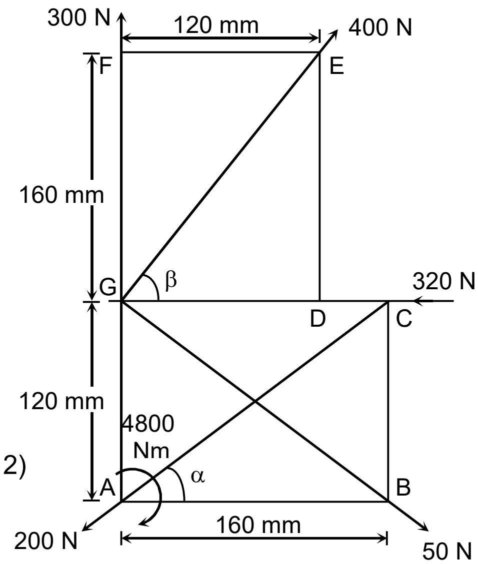
$=$ 200 N ($\leftarrow$)

negative sign indicates leftward direction

$\sum F_Y = 300 + 400\sin\beta - 200\sin\alpha - 50\sin\alpha$

$=$ 470 N ($\uparrow$)

positive sign indicates upward direction.



The resultant magnitude is given by

$$R = \sqrt{\sum F_X^2 + \sum F_Y^2} = 510.78 \text{ N}$$

The direction of the resultant is given by

$\theta = \tan^{-1}(\sum F_Y / \sum F_X)$

$= \tan^{-1}(470 / 200)$

$= 66.95°$ ($\measuredangle$)

To find the position of resultant,

Consider point A as the origin.

$\sum M_A = -4800 - 50\sin\alpha\,(0.16) + 320\,(0.12) - 400\cos\beta\,(0.12)$

$= 0$

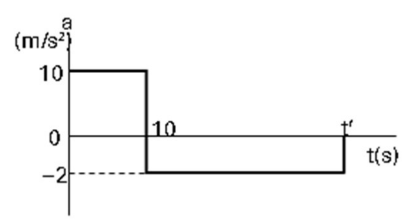
The x intercept is given by,

$$x = \frac{\sum M_A}{\sum F_Y} = 0$$

This means that the resultant cuts the x axis

(line AB at x = 0, i.e. through the origin, point A)

II) A car starts from rest and travels along a straight track. It accelerates at a constant rate for 10 seconds, then immediately begins to decelerate at a constant rate and it comes to rest at t'=60 sec. Draw the velocity-time (v-t) for this motion. **04**

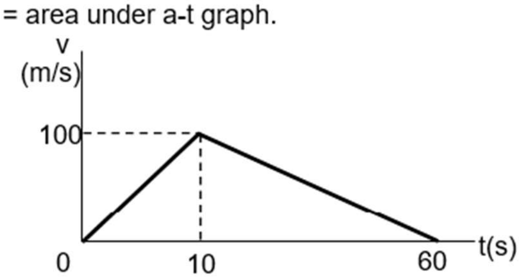


The car starts from rest. Hence $v_0 = 0$ and $s_0 = 0$

Change in velocity in a time interval = area under a-t graph.

$v_{10} - v_0 = (10 \times 10) = 100$ m/s

$\therefore \quad v_{10} = v_0 + 100 = 100$ m/s

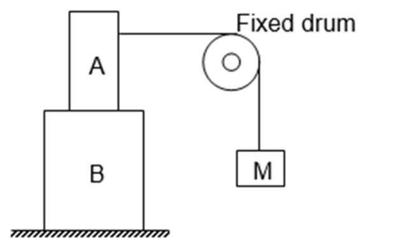
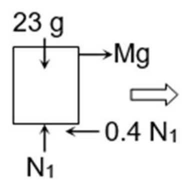
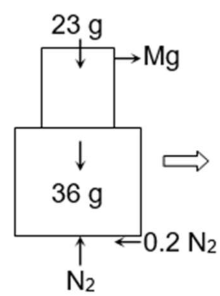
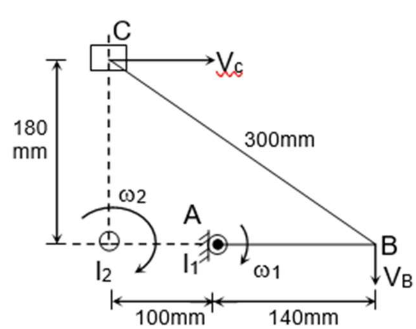


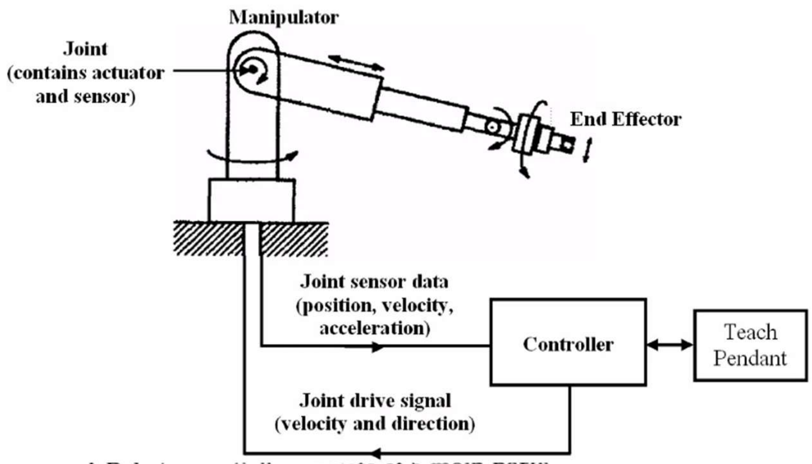
Velocity at time $t' = 0$

$\therefore \quad 0 - v_{10} = -[(t' - 10)(2)]$

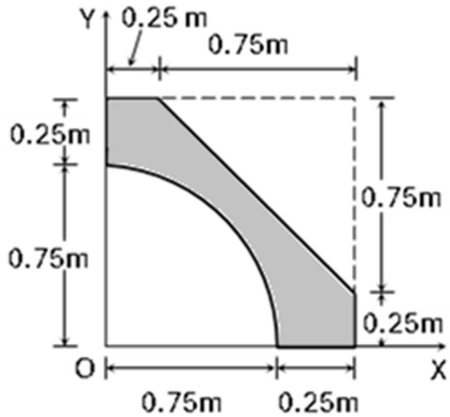
$\therefore \quad -100 = -[2t' - 20]$

$\therefore \quad t' = 60$ seconds

B.	I) The mass of A is 23 Kg and mass of B is 36 Kg. The coefficient of friction are 0.4 between A and B, and 0.2 between ground and block B. Assume smooth drum. Determine the maximum mass of M at impending motion.  The motion can impend in 2 ways (i) Only A starts motion or (ii) Both start together Case 1: Only A starts motion $\Sigma F_y = 0 \quad \therefore \quad N_1 = 23g\ N(\uparrow)$ $\Sigma F_x = 0 \quad \therefore \quad Mg - 0.4\ N_1 = 0$ $M = 9.2$ kg  Case 2: Both A and B start motion together $\Sigma F_y = 0 \quad \therefore \quad N_2 = 59g\ N(\uparrow)$ $\Sigma F_x = 0 \quad \therefore \quad Mg - 0.2\ N_2 = 0$ $M = 11.8$ kg  As "M" is lesser in case (1) so case (1) will start $M = 9.2$ kg	06
	II) The car moves in a straight line such that for a short time its velocity is defined by $v = (9t^2 + 2t)$m/s. Where t is in seconds. Determine its position and acceleration when t = 3 sec. **Given:** $[V = 9t^2 + 2t]$m/s $\therefore \quad S = \int v dt = 3t^3 + t^2 + C_1$ Assuming at t = 0, S = 0 (i.e position = displacement) We get $C_1 = 0$ $\therefore \quad [S = 3t^3 + t^2]$ m and $[a = \frac{dv}{dt} = 18t + 2]$ m/s² at $t = 3$ sec $S_3 = 90$ m $a_3 = 56$ m/s²	04
C.	I) In figure collar C slides on a horizontal rod. In the position shown rod AB is horizontal and has angular velocity of 0.6 rad/sec clockwise. Determine angular velocity of BC and velocity of collar C.  Given : $\omega_1 = 0.6$ r/s (↻) Rod AB : $V_B = l(I_1B) \times \omega_1$ $= 140 \times 0.6$ $= 84$ mm/s (↓) Rod BC : $\omega_2 = \frac{V_B}{l(I_2B)} = \frac{84}{240}$	06

	$= 0.35$ r/s (↻) $V_C = l(I_2C) \times \omega_2$ $= 180 \times 0.35$ $= 63$ mm/s (→)	
	II) Describe various parts of Robot with neat sketch.  A Robot essentially consists of 5 main parts 1. **Manipulator** - It makes up the main structure of the robot. It consists of links and joints and gives the shape and form to a robot. Joints in a robot are mainly of two kinds viz. Revolute and Prismatic. A revolute joint allows only rotation and does not allow any linear motion at the joint. Whereas a Prismatic joint allows linear motion at the joint and does not facilitate any rotation. 2. **End effector** - It is the last extreme part of a robot. It is designed to interact with the environment. It is finally the end effector which performs the task for which the robot is designed. The type of end effector depends on the task to be performed. For example, for a robot designed for picking an object from one point and placing at some other point, the end effector would be a gripper, which should hold and also release the object as desired. 3. **Actuators** - Actuators are devices which provide motion to the joints and links. Actuators convert either electrical, air or hydraulic energy into mechanical energy. Actuators may be servo motors or pneumatic actuators or hydraulic actuators. 4. **Sensors** - Sensors monitor the robot's internal and external environment. Sensors track the position, orientation, speed, acceleration, and other changes and sends the same as input signal to the controller for adjusting robot's movement. 5. **Controller** - It is the brain of a robot. It receives command inputs from the computer and signals from the sensors. On these bases it directs and controls the different actuators placed at different joints for the designed motion.	04

D. I) Find the centroid shaded area. 08



For square:

$A_1 = 1 \times 1 = 1\ m^2$

$x_1 = 0.5$ m

For triangle:

$A_2 = -1/2 \times 0.75 \times 0.75 = -0.281 m^2$

$x_2 = 1 - 0.75/3 = 0.75$ m

For quartercircle:

$A_3 = -\pi(0.75)^2 / 4 = -0.441\ m^2$

$x_3 = 4(0.75)/(3\pi) = 0.318$ m

A summary of these calculations are given in the table below.

Part	**A_i**	**x_i**	**$A_i x_i$**
Square	1	0.5	0.5
Triangle	–0.281	0.75	–0.211
Quartercircle	–0.441	0.318	–0.140
	$\sum A_i = 0.278$		**$\sum A_i x_i = 0.149$**

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = 0.536 \text{ m}$$

We can see that the angle bisector of the two coordinate axes is the axis of symmetry for the given area. Since the centroid has to lie on the axis of symmetry, hence the x–coordinate and y–coordinate of the centroid must be identical.

Hence, $\bar{y} = \bar{x} = 0.536$ m

Hence the coordinates of centroid are (0.536, 0.536) in m.

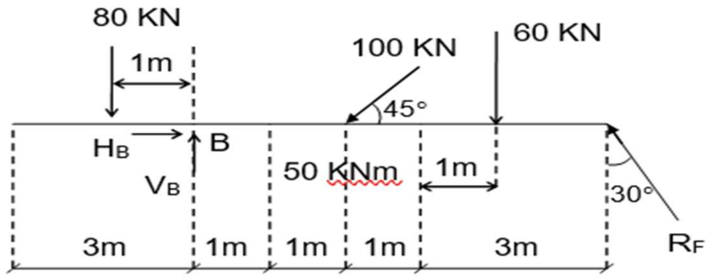
II) Define Angle of Repose and Angle of Friction 02

- **Angle of Repose (θ)**

If a block is placed on a rough inclined plane and if the inclination of the plane is gradually increased, then the angle θ at which the block would have impending motion down the slope is called the angle of repose.

- **Angle of Friction**

The angle made by the resultant R with the normal to the surface of contact when the body has impending motion is called the angle of friction.

<table>
<tr><td>E.</td><td>Find the support reactions for the beam loaded and supported as shown in fig.


$\Sigma M_B = 0$ (↺ positive)
∴ $80 \times 1 - 50 - 100 \sin 45 \times 2$
$- 60 \times 4 + R_F \cos 30 \times 6 = 0$
$R_F = 67.63$ KN (↖)

$\Sigma F_x = 0$ (→ positive)
∴ $H_B - 100 \cos 45 - R_F \sin 30 = 0$
$H_B = 104.53 = 104.53$ KN (→)

$\Sigma F_y = 0$ (↑ positive)
∴ $- 80 + V_B - 100 \sin 45 - 60 + R_F \cos 30 = 0$
$V_B = 152.14 = 152.14$ KN (↑)</td><td>10</td></tr>
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<table>
<tr><td>Q.3</td><td>Attempt any THREE. (All questions carry equal marks)</td><td>15</td></tr>
<tr><td>A.</td><td>Write a Homogeneous matrix that represents Pure Rotation about all the 3 axes

Projection of z_1 on z = 1

Putting this vector data in matrices form, we have, Rotation Matrix about z axis as $R_{OT}(z)$
$$R_{OT}(z) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (6)$$
Similarly, Rotation Matrix about x axis can be obtained as $R_{OT}(x)$
$$R_{OT}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (7)$$
Similarly, Rotation Matrix about y axis can be obtained as $R_{OT}(y)$
$$R_{OT}(y) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (8)$$</td><td>05</td></tr>
<tr><td>B.</td><td>A rod AB, 26m long leans against a vertical wall. The end A on the floor is drawn away from the wall at a rate of 24 m/s, when the end A of the rod is 10m from the wall, determine the velocity of end B sliding down vertically and the angular velocity of rod AB.</td><td>05</td></tr>
</table>

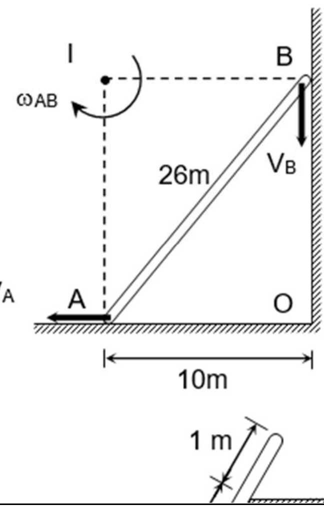
Given V_A = 24m/s (←)
To find : V_B , ω_{ab}

Velocity directions of points A and B are as shown in the figure. Hence we can locate the ICR by drawing perpendiculars to V_A and V_B.
By Pythagoras theorem

$$BO = \sqrt{26^2 - 10^2} = 24\text{m}$$

$$V_A = IA \times \omega_{AB}$$
$$\therefore \quad 24 = 24 \times \omega_{AB}$$
$$\therefore \quad \omega_{AB} = 1 \text{ r/s (clockwise)}$$

$$V_B = IB \times \omega_{AB}$$
$$= 10 \times 1$$
$$= 10 \text{ m/s } (\downarrow)$$

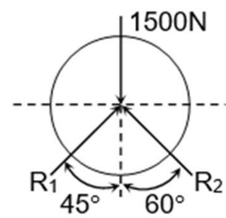


C. A cylinder with 1500 N weight is resting in an unsymmetrical smooth groove as shown in figure. Determine the reactions at the points of contacts. — 05

By Lami's theorem

$$\frac{-1500}{\sin(45+60)} = \frac{-R_1}{\sin(180-60)} = \frac{-R_2}{\sin(180-45)}$$

R_1 = 1344.86 N (↗) and R_2 = 1098.08 N (↖)



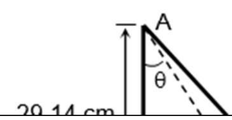
D. Find the centroid for given lamina — 05

To find the centroid of the lamina.

Part	A_i	x_i	y_i	A_ix_i	A_iy_i
Triangle AEB	450	10	22	4500	9900
Rectangle ECDO	600	25	6	15000	3600
	ΣA_i = 1050			ΣA_ix_i = 19500	ΣA_iy_i = 13500

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = 18.57 \text{ cm}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = 12.86 \text{ cm}$$



*********** All the Best***********