(Affiliated to University of Mumbai)	***End Semester Examination (R-24/R-25) SH 2026 Answer Key with marking scheme***
Branch: CE/AIDS/AIML/CSE	Course: Applied Mathematics III
Year/ Semester: SE / III	Course code: CEC301/AIDSC301/AIMLC301/CSEC301
Time: 03 hours	Marks: 80

		Marks
Q. 1	All questions are compulsory.	
A.	$f(t) = t\,e^{-2t}\sin^2 t$, $g(t) = \sin^2 t = \frac{1-\cos 2t}{2}$, $L\{\sin^2 t\} = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$, $L\{t\sin^2 t\} = -\frac{d}{ds}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$, $= -\frac{1}{2}\left[-\frac{1}{s^2} - \left(\frac{s^2+4-s\cdot 2s}{(s^2+4)^2}\right)\right]$, $= \frac{1}{2}\left[\frac{1}{s^2} + \frac{4-s^2}{(s^2+4)^2}\right]$ ✓ $L\{e^{-2t}\cdot t\sin^2 t\} = \frac{1}{2}\left[\frac{1}{(s+2)^2} + \frac{4-(s+2)^2}{[(s+2)^2+4]^2}\right]$ #	
B.	$I = \int_0^\infty e^{-t}\left(\frac{\sin 2t + \sin 3t}{t}\right)dt = L\left\{\frac{\sin 2t+\sin 3t}{t}\right\}_{s=1}$ ① $L\{\sin 2t + \sin 3t\} = \frac{2}{s^2+4} + \frac{3}{s^2+9}$ $L\left\{\frac{\sin 2t+\sin 3t}{t}\right\} = \int_s^\infty \left(\frac{2}{s^2+4} + \frac{3}{s^2+9}\right)ds$ $= \tan^{-1}(s/2) + \tan^{-1}(s/3)\Big]_s^\infty$ $= \left(\frac{\pi}{2}+\frac{\pi}{2}\right) - \left(\tan^{-1}(s/2)+\tan^{-1}(s/3)\right)$ $= \pi - \left[\tan^{-1}\left(\frac{\frac{s}{2}+\frac{s}{3}}{1-\frac{s}{2}\cdot\frac{s}{3}}\right)\right]$ $= \pi - \tan^{-1}\left[\frac{5s/6}{(6-s^2)/6}\right]$ $= \pi - \tan^{-1}\left(\frac{5s}{6-s^2}\right)$ put $s=1$ $I = \pi - \tan^{-1}\left(\frac{5}{5}\right)$ $= \pi - \frac{\pi}{4}$ $= \frac{3\pi}{4}$ #	
C.	Sol. : Let $f(x) = a_0 + \sum a_n \cos nx \quad [\because l = \pi]$ $a_0 = \frac{1}{\pi}\int_0^\pi f(x)\,dx = \frac{1}{\pi}\int_0^\pi (\pi x - x^2)\,dx = \frac{1}{\pi}\left[\frac{\pi x^2}{2} - \frac{x^3}{3}\right]_0^\pi = \frac{\pi^2}{6}$ $a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^\pi (\pi x - x^2)\cos nx\,dx$ $= \frac{2}{\pi}\left[(\pi x - x^2)\cdot\frac{\sin nx}{n} - (\pi - 2x)\cdot\left(-\frac{\cos nx}{n^2}\right) + (-2)\left(-\frac{\sin nx}{n^3}\right)\right]_0^\pi$ $\therefore\ a_n = -2\left(\frac{1+\cos n\pi}{n^2}\right) = \begin{cases} 0 & \text{when } n \text{ is odd} \\ -4/n^2 & \text{when } n \text{ is odd} \end{cases}$ [By § 5, page 3-1($\therefore\ x(\pi - x) = \frac{\pi^2}{6} - 4\left[\frac{\cos 2x}{2^2} + \frac{\cos 4x}{4^2} + \frac{\cos 6x}{6^2} + \ldots\right]$ $= \frac{\pi^2}{6} - \left[\frac{1}{1^2}\cos 2x + \frac{1}{2^2}\cos 4x + \frac{1}{3^2}\cos 6x + \ldots\right]$	
D.	$\frac{\partial u}{\partial x} = 2qxy + 4x$, $\frac{\partial u}{\partial y} = qx^2 + 3ry^2 - 4y$, $\frac{\partial v}{\partial x} = -3px^2 + 4y + 3y^2$, $\frac{\partial v}{\partial y} = 4x + 6xy$ $2q = 6 \implies \mathbf{q = 3}$ $q = 3p \implies 3 = 3p \implies \mathbf{p = 1}$ $\mathbf{p = 1,\quad q = 3,\quad r = -1}$	

E.

$\bar{x} = 31$, $\bar{y} = 25$ (approx)

x	$dx = x-31$	dx^2	y	$dy = y-26$	dy^2	$dx\,dy$
23	−8	64	18	−8	64	64
27	−4	16	22	−4	16	16
28	−3	09	23	−3	9	9
29	−2	4	24	−2	4	4
30	−1	1	25	−1	1	1
31	0	0	26	0	0	0
33	2	4	28	2	4	4
35	4	16	29	3	9	12
36	5	25	30	4	16	20
39	8	64	32	6	36	48
	1	203		−3	159	178

$$\therefore r = \frac{178 - [(1\times -3)/10]}{\sqrt{203 - \frac{1}{10}}\sqrt{159 - \frac{9}{10}}} = \frac{178.3}{\sqrt{202.9 \times 158.1}} = 0.99$$

F.

i) $\Sigma p_i = 1 \Rightarrow \frac{1}{8} + m + n + \frac{1}{4} + \frac{1}{12} = 1 \Rightarrow \boxed{m+n = \frac{13}{24}}$ — (1)

$\mu_1' = \text{Mean} = \Sigma p_i x_i = 16 \Rightarrow 1 + 12m + 16n + 5 + 2 = 16$

$\Rightarrow \boxed{3m + 4n = 2}$ — (2)

Solving (1) & (2); $\boxed{m = 1/6}$, $\boxed{n = 3/8}$

$\mu_2' = \Sigma p_i x_i^2 = 8 + 144m + 256n + 100 + 48$

$= 276$

$\therefore$ Variance $= \mu_2' - \mu_1'^2 = 276 - (16)^2 = 276 - 256$

$= 20$

Q.2A

$$d(s) = \frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$2s^2 - 6s + 5 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$$

$s = 1$: $1 = A(-1)(-2) \Rightarrow A = 1/2$

$s = 2$; $1 = B(-1) \Rightarrow B = -1$

$s = 3$; $5 = 2C \Rightarrow C = 5/2$

$$\phi(s) = \frac{1}{2}\cdot\frac{1}{s-1} - \frac{1}{s-2} + \frac{5}{2}\cdot\frac{1}{s-3}, \quad \text{ILT}$$

$$f(t) = \frac{1}{2}\cdot e^t - e^{2t} + \frac{5}{2}\cdot e^{3t}$$ ##

OR

Q.2B.

Now, $L(y'') = s^2\bar{y} - sy(0) - y'(0) = s^2\bar{y} - s$ and $L(t) = \frac{1}{s^2}$.

$\therefore$ The equation (1) becomes

$$s^2\bar{y} - s + \bar{y} = \frac{1}{s^2} \quad \therefore \quad s^2\bar{y} + \bar{y} = s + \frac{1}{s^2} = \frac{s^3+1}{s^2}$$

$$\therefore (s^2+1)\bar{y} = \frac{s^3+1}{s^2} \quad \therefore \quad \bar{y} = \frac{s^3+1}{s^2(s^2+1)}$$

Let $\frac{s^3+1}{s^2(s^2+1)} = \frac{a}{s} + \frac{b}{s^2} + \frac{cs+d}{s^2+1}$

$$\therefore s^3 + 1 = as(s^2+1) + b(s^2+1) + (cs+d)s^2$$
$$= (a+c)s^3 + (b+d)s^2 + as + b$$

Equating like powers of s,

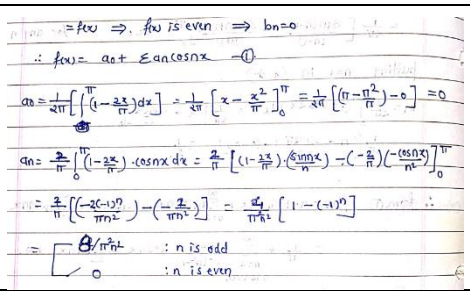
$a + c = 1$, $b + d = 0$, $a = 0$, $b = 1$ $\quad \therefore \quad a = 0$, $b = 1$, $c = 1$, $d = -1$

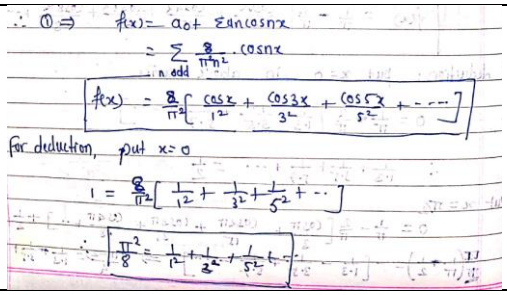
$$\therefore \bar{y} = \frac{1}{s^2} + \frac{s-1}{s^2+1} = \frac{1}{s^2} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

Taking inverse Laplace transform

$$y = L^{-1}\left(\frac{1}{s^2}\right) + L^{-1}\left(\frac{s}{s^2+1}\right) - L^{2}\left(\frac{1}{s^2+1}\right) = t + \cos t - \sin t.$$

Q.3A





OR

Q.3B

$l = \pi$

$$a_0 = \frac{1}{\pi}\cdot\frac{1}{12}\int_0^{2\pi}(3x^2-6x\pi+2\pi^2)\,dx = \frac{1}{12\pi}\left[x^3-3x^2\pi+2\pi^2x\right]_0^{2\pi} = \frac{1}{12\pi}\left[8\pi^3-12\pi^3+4\pi^3\right] = 0$$

$$a_n = \frac{1}{12\pi}\int_0^{2\pi}(3x^2-6x\pi+2\pi^2)\cdot\cos nx\,dx = \frac{1}{12\pi}\left[(3x^2-6x\pi+2\pi^2)\left(\frac{\sin nx}{n}\right)-(6x-6\pi)\left(\frac{-\cos nx}{n^2}\right)+(6)\left(\frac{-\sin nx}{n^3}\right)\right]_0^{2\pi}$$

0×0

0×0

$$= \frac{1}{12\pi}\left[\left(\frac{6\pi}{n^2}\right)-\left(-\frac{6\pi}{n^2}\right)\right] = \frac{1}{n^2}$$

$$b_n = \frac{1}{12\pi}\int_0^{2\pi}(3x^2-6x\pi+2\pi^2)\sin nx\,dx = \frac{1}{12\pi}\left[(3x^2-6x\pi+2\pi^2)\left(\frac{-\cos nx}{n}\right)-(6x-6\pi)\left(\frac{-\sin nx}{n^2}\right)+(6)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}\times\times$$

$$= \frac{1}{12\pi}\left[\left[(12\pi^2-12\pi^2+2\pi^2)\left(\frac{-1}{n}\right)+6\frac{1}{n^3}\right]-\left[2\pi^2\left(\frac{-1}{n}\right)+\frac{6}{n^3}\right]\right]$$

$$= \frac{1}{12\pi}\left[-\frac{2\pi^2}{n}+\frac{6}{n^3}+\frac{2\pi^2}{n}-\frac{6}{n^3}\right] = 0$$

① $\Rightarrow$ $f(x) = 0+\sum\frac{1}{n^2}\cdot\cos nx + c$

$$\frac{1}{12}(3x^2-6x\pi+2\pi^2) = \frac{1}{1^2}\cos x+\frac{1}{2^2}\cos 2x+\frac{1}{3^2}\cos 3x+\cdots$$

$x = 0$

$$\frac{\pi^2}{6} = 1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$

Q.4A

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2} = e^x\sin y - e^x\sin y \qquad \boxed{f'(z) = e^z} \qquad f(z) = e^x\cos y + ie^x\sin y + C$$

OR

Q.4B

$u-v = (x-y)(x^2+4xy+y^2) = U$: real part.

$= x^3+4x^2y+xy^2-x^2y-4xy^2-y^3$

$U = x^3+3x^2y-3xy^2-y^3$

$U_x = 3x^2+6xy-3y^2 \xrightarrow[y=0]{x=z} \boxed{d_1 = 3z^2}$

$U_y = 3x^2-6xy-3y^2 \xrightarrow[y=0]{x=z} \boxed{d_2 = 3z^2}$

By M-T method

$(1+i)f'(z) = d_1 - i d_2$

$= 3z^2 - i\,3z^2$

$= (1-i)3z^2$

$(1+i)f(z) = (1-i)\cdot z^3 + c'$

$f(z) = \frac{1-i}{1+i}z^3 + c$

$\boxed{f(z) = -iz^3+C}$

Q.5A

x	y	R_x	R_y	$d_i = R_x - R_y$	d_i^2
32	40	8	6	2	4
55	(30)	2	7·5	−5·5	30·25
(49)	70	3·5	2	1·5	2·25
60	20	1	10	−9	81
(43)	(30)	5·5	7·5	−2	4
37	50	7	4	3	9
(43)	72	5·5	1	4·5	20·25
(49)	60	3·5	3	0·5	0·25
10	45	10	5	5	25
20	25	9	9	0	0
$n=10$					$\Sigma d_i^2 = 176$

$m_1 = 2,\ m_2 = 2,\ m_3 = 2$

$\frac{1}{12}(m_1^3-m_1)+\frac{1}{12}(m_2^3-m_2)+\frac{1}{12}(m_3^3-m_3)$

$= \frac{1}{12}[6+6+6] = \frac{18}{12} = \frac{3}{2} = 1{\cdot}5$

$\therefore\ R = 1-\left[\frac{6[176+1{\cdot}5]}{990}\right]$

$= \boxed{-0{\cdot}0758}$ #

OR

Q.5B

i) y on x : $\bar{x} = 65$ $\quad \bar{y} = 66$

$$b_{yx} = \frac{45604 - \left[\frac{650 \times 660}{10}\right]}{47648 - \left[\frac{(650)^2}{10}\right]} = 0.5009$$

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 66 = 0.5009(x - 65)$

$\boxed{y = 0.5009x + 33.44}$

$$b_{xy} = \frac{45604 - \left[\frac{650 \times 660}{10}\right]}{45784 - \frac{660^2}{10}} = 1.2158$$

ii) x on y

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$x - 65 = 1.2158(y - 66)$

$\boxed{x = 1.2158y - 15.2428}$

Q.6A

i) $\int f(x)dx = 1 \rightarrow \int_0^\infty kxe^{-x/3}dx = 1$

$k\left[x\left(\frac{e^{-x/3}}{-1/3}\right) - 1\left(\frac{e^{-x/3}}{1/9}\right)\right]_0^\infty = 1$

$k\left[[0] - [0 - 9]\right] = 1 \rightarrow 9k = 1$

$\Rightarrow \boxed{k = \frac{1}{9}}$

ii) $E(x) = \text{Mean} = \int x f(x)dx$.

$= k\int_0^\infty x^2 e^{-x/3}dx = k\left[x^2\left(\frac{e^{-x/3}}{-1/3}\right) - (2x)\left(\frac{e^{-x/3}}{1/9}\right) + 2\left(\frac{e^{-x/3}}{-1/27}\right)\right]_0^\infty$

$= \frac{1}{9}\left[[0] - [-54]\right] = \frac{54}{9} = \boxed{6 = \mu_1'}$

iii) $P(X > 6) = \int_6^\infty kxe^{-x/3}dx$

$= \frac{1}{9}\left[x\left(\frac{e^{-x/3}}{-1/3}\right) - \left(\frac{e^{-x/3}}{1/9}\right)\right]_6^\infty$

$= \frac{1}{9}\left[[0] - [-18e^{-2} - 9e^{-2}]\right] = \frac{27e^{-2}}{9} = \frac{3}{e^2}$

$\boxed{P(X > 6) = 0.406}$

OR

Q.6B

We have;

x:	1	2	3	4	5	6
p:	1/6	1/6	1/6	1/6	1/6	1/6

$M_0(t) = \sum p_i e^{tx_i} = \frac{1}{6}\left[e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]$

$\mu_1' = \frac{d}{dt}\left[M_0(t)\right]_{t=0} = \frac{1}{6}\left[e^t + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}\right]_{t=0}$

$= \frac{1 + 2 + 3 + 4 + 5 + 6}{6} = \frac{21}{6} = \frac{7}{2}$

$\mu_2' = \frac{d^2}{dt^2}\left[M_0(t)\right] = \frac{1}{6}\left[e^t + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t}\right]$

$= \frac{1}{6}\left[1 + 4 + 9 + 16 + 25 + 36\right]$

$= \frac{91}{6}$

$$\mu_3' = \frac{1}{6}(1 + 8 + 27 + 64 + 125 + 216) = \frac{441}{6} = \frac{147}{2} = \mathbf{73.5}$$

$$\mu_4' = \frac{1}{6}(1 + 16 + 81 + 256 + 625 + 1296) = \frac{2275}{6} \approx \mathbf{379.167}$$

*********** All the Best***********