Ans Key of DSGT Set II
ESE (R-24) SH2025

Space for Question No

Q 1 A) $x \equiv y \pmod 7$ $\rightarrow$ 7 divides $x-y$

Now $x-x=0$ and 7 divides 0

$\therefore (x,x) \in R$ $\therefore$ R is reflexive — 1 Mark

Consider $(x,y) \in R$

$\therefore x \equiv y \pmod 7$ $\therefore$ 7 divides $x-y$

$\therefore x-y=7m$ for some $m \in Z$

$\therefore y-x=-7m=7\times(-m)$, where $-m \in Z$

$\therefore y \equiv x \pmod 7$

$\therefore (y,x) \in R$ $\therefore$ R is symmetric — 2 Marks

Consider $(x,y) \in R$ and $(y,z) \in R$

$\therefore x \equiv y \pmod 7$ and $y \equiv z \pmod 7$

$\therefore$ 7 divides $x-y$ and 7 divides $y-z$

$\therefore x-y=7m$ and $y-z=7n$, for some $m,n \in Z$

$\therefore x-y+y-z=7m+7n$

$\therefore x-z=7(m+n)$, where $m+n \in Z$

$\therefore x \equiv z \pmod 7$

$\therefore$ R is transitive $\therefore$ R is equivalence — 2 Marks

B) Let $P(n)$ be $1^2+2^2+\cdots+n^2=\frac{n}{6}(n+1)(2n+1)$

Step-I :- Putting $n=1$, we get

LHS $=1$, RHS $=\frac{1}{6}(1+1)(2+1)=1$

$\therefore P(1)$ is true. — 1 Mark

Step-II :- Assume that $P(K)$ is true.

$\therefore 1^2+2^2+\cdots+K^2=\frac{K}{6}(K+1)(2K+1)$

Now $1^2+2^2+\cdots+(K+1)^2$

$=1^2+2^2+\cdots+K^2+(K+1)^2=\frac{K}{6}(K+1)(2K+1)+(K+1)^2$

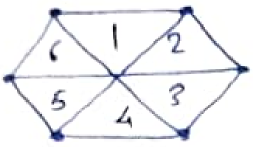
$=\frac{(K+1)(K+2)(2K+3)}{6}$ $\therefore P(K+1)$ is true — 3 Marks

Step III By mathematical induction, $P(n)$ is true, $\forall n \in N$ — 1 Mark

c) If n pigeons are assigned to m pigeonholes and $m < n$, then at least one pigeonhole contains two or more pigeons.

1 Mark

Divide hexagon in 6 equilateral triangles of sides one unit as shown in fig.



$\therefore$ there are 6 pigeonholes and 7 points are 7 pigeons.

$\therefore$ at least 2 points will lie in the same subtriangle

$\therefore$ Distance between those 2 points is less than 1 unit

4 Marks

D) Consider $A \in P(S)$. Now $A \subseteq A$

$\therefore ARA$ $\therefore$ R is reflexive 1 Mark

Consider $A \subseteq B$ ~~$\in P(S)$~~ and $B \subseteq A$

Then $A = B$ $\therefore$ R is antisymmetric 2 Marks

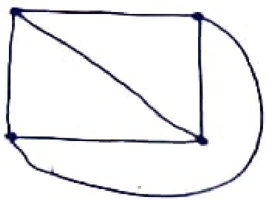
Consider $A \subseteq B$ and $B \subseteq C$ $\therefore A \subseteq C$

$\therefore$ R is transitive $\therefore$ R is transitive

$\therefore$ R is partial order relation 2 Marks

Here R is subset relation

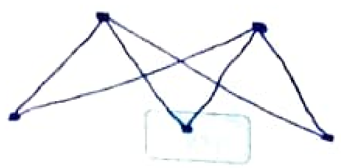
E) The diagram of a graph drawn in such a way that no two edges intersect each other is called as planar graph.



2 Marks

$G = (V, E)$ is complete bipartite, if

1) G is bipartite, 2) G is simple, 3) $V = U \cup W$ is partition of V, then each vertex $u_i \in U$ is joined to every vertex $v_j \in W$.



Question No

F) Let G - Set of rational numbers.

Consider $a * b = a + b - ab$, where $a, b \in G$

$\because$ addition, subtraction and multiplication of rational numbers is again a rational number,

$\therefore$ $a * b$ is a rational number.

$\therefore$ $*$ is binary — 1 Mark

TPT $a*(b*c) = (a*b)*c$

$LHS = a*(b*c) = a*(b+c-bc)$

$= a * e$, where $e = b+c-bc$

$= a + e - ae$

$= a + b + c - bc - a(b+c-bc)$

$= a+b+c-bc-ab-ac+abc$

$RHS = (a*b)*c = (a+b-ab)*c$

$= d * c$, where $d = a+b-ab$

$= d + c - dc = a+b-ab+c-(a+b-ab)c$

$= a+b-ab-ac-bc+abc+c = LHS$

$\therefore$ $*$ is associative — 2 Marks

Consider $a * e = a$

$\therefore$ $a + e - ae = a$

$\therefore$ $e - ae = 0$

$\therefore$ $e(1-a) = 0$

$\therefore$ $e = 0$ is identity — 1 Mark

Consider $a * b = e$

$\therefore$ $a + b - ab = 0$

$\therefore$ $b - ab = a$

$\therefore$ $b(1-a) = a$

$\therefore$ $b = \dfrac{a}{1-a}$, $a \neq 1$ is inverse of a — 1 Mark

$\therefore$ $(G, *)$ is a group.

Space for Marks | Question No.

Q2 A)

$$W_0 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$ 1 Mark

Step-1 :-

Column 1 → pi = 1, 3 , Row 1 → qj = 1, 2, 4

∴ We put 1 in (pi, qj) i.e. (1, 1), (1, 2), (1, 4), (3, 1), (3, 2), (3, 4) positions.

$$\therefore W_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$ 2 Marks

Step-2 :-

Column 2 → pi = 1, 3, 4 , Row 2 → qj = 4

∴ We put 1 in (pi, qj) i.e. (1, 4), (3, 4), (4, 4) positions

$$\therefore W_2 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$ 2 Marks

Step-3 :-

Column 3 → pi = 4 , Row 3 → qj = 1, 2, 4

∴ We put 1 in (pi, qj) i.e. (4, 1), (4, 2), (4, 4) positions.

$$\therefore W_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ 2 Marks

Step-4 :- Since 1 is present in all places of column 4 and row 4,

$$\therefore W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

∴ Transitive clousure = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}

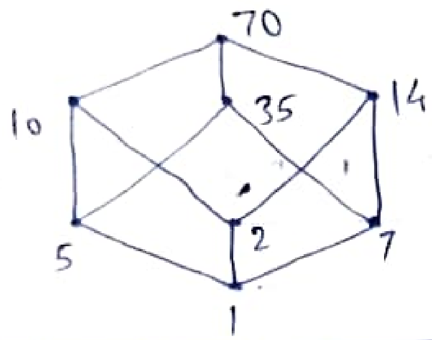
2 Marks

1 Mark

Q2 B) A poset L in which every pair $\{a, b\}$ of two elements of L has LUB and GLB both is Lattice. 1 Mark

Set of all divisions of 70

$= D_{70} = \{1, 2, 5, 7, 10, 14, 35, 70\}$ 1 Mark



1 Mark

V	1	2	5	7	10	14	35	70
1	1	2	5	7	10	14	35	70
2	2	2	10	14	10	14	70	70
5	5	10	5	35	10	70	70	70
7	7	14	35	7	70	14	35	70
10	10	10	10	70	10	70	70	70
14	14	14	70	14	70	14	70	70
35	35	35	70	35	70	70	35	70
70	70	70	70	70	70	70	70	70

3 Marks

Λ	1	2	5	7	10	14	35	70
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
5	1	1	5	1	5	1	5	5
7	1	1	1	7	1	7	7	7
10	1	2	5	1	10	2	5	10
14	1	2	1	7	2	14	7	14
35	1	1	5	7	5	7	35	35
70	1	2	5	7	10	14	35	70

3 Marks

$\because$ every pair of elements has LUB and GLB both

$\therefore D_{70}$ is lattice. 1 Mark

Q2 c) A ring is an ordered triplet $(R, +, \cdot)$ where R is a non-empty set and $+$ and $\cdot$ are two binary operations on R satisfying following axioms.

1) $(R, +)$ is commutative group
2) $\cdot$ is associative
3) $\cdot$ distributes over $+$.

1 Mark

$\oplus$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$\otimes$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

4 Marks

$\oplus$ is binary and associative
0 is identity

Element	0	1	2	3	4	5
Inverse	0	5	4	3	2	1

$\oplus$ is commutative
$\therefore$ $(R, \oplus)$ is Abelian group. 3 Marks

$\otimes$ is associative

$\oplus$ and $\otimes$ are distributive over each other.
$\therefore$ $(R, \oplus, \otimes)$ is a ring 2 Marks

Q2 D) S = Set of all integers between 1 and 500

$n(S) = 50$

A : Set of all integers in S divisible by 3

$n(A) = 166$

B = Set of all integers in S divisible by 5

$n(B) = 100$

C = Set of all integers in S divisible by 7

$n(C) = 71$

$A \cap B$ = Set of all integers in S divisible by 15

$n(A \cap B) = 33$

$B \cap C$ = Set of all integers in S divisible by 35

$n(B \cap C) = 14$

$A \cap C$ = Set of all integers in S divisible by 21

$n(A \cap C) = 23$

$A \cap B \cap C$ = Set of all integers in S divisible by 105

$n(A \cap B \cap C) = 4$

4 Marks

i) $n(B \cap C \cap A') = n(B \cap C) - n(A \cap B \cap C)$

$= 14 - 4 = 10$ 2 Marks

ii) $n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$

$= 33 - 4 = 29$ 2 Marks

iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 166 + 100 + 71 - 33 - 14 - 23 + 4 = 271$

$\therefore n((A \cup B \cup C)') = n(S) - n(A \cup B \cup C)$

$= 500 - 271 = 229$ 2 Marks

Q 2 E)

⊕	0000000	0010110	0101000	0111110	1000101	1010011	1101101	1111011
0000000	0000000	0010110	0101000	0111110	1000101	1010011	1101101	1111011
0010110	0010110	0000000	0111110	0101000	1010011	1000101	1111011	1101101
0101000	0101000	0111110	0000000	0010110	1101101	1111011	1000101	1010011
0111110	0111110	0101000	0010110	0000000	1111011	1101101	1010011	1000101
1000101	1000101	1010011	1101101	1111011	0000000	0010110	0101000	0111110
1010011	1010011	1000101	1111011	1101101	0010110	0000000	111110	0101000
1101101	1101101	1111011	1000101	1010011	0101000	0111110	000000	0010110
1111011	1111011	1101101	1010011	1000101	0111110	0101000	0010110	0000000

6 Marks

⊕ is binary and associative 1 Mark

0000000 is identity 1 Mark

every element is its own inverse. 1 Mark

$\therefore N = \{0000000, 0010110, 0101000, 0111110, 1000101, 1010011, 1101101, 1111011\}$

is a subgroup of B^7

$\therefore$ e is a group code. 1 Mark

F) Two graphs $G=(V,E)$ and $G'=(V',E')$ are isomorphic if there exists one-one correspondence $f: V \to V'$ such that if $v_1, v_2 \in V$ and $v_1', v_2' \in V'$ with $f(v_1) = v_1'$ and $f(v_2) = v_2'$, then number of edges between v_1 and v_2 is same as those between v_1' and v_2'. The function f is isomorphism between G and G'. 2 Marks

We first see that both graphs G_1 and G_2 have same number of vertices (6) and same number of edges (9).

In G_1 and G_2, all vertices are of degree 3.

Consider the correspondence

$v_1 \to u_1$, $v_2 \to u_2$, $v_3 \to u_3$, $v_4 \to u_4$, $v_5 \to u_5$, $v_6 \to u_6$

$\therefore$ G_1 and G_2 are isomorphic 8 Marks

Q3 A) If $f: X \to Y$ and $g: Y \to Z$ are two functions such that domain of g and codomain of f are same, then a function $h: X \to Z$ is composition of f and g.

1 Mark

$g \circ f \circ h(x) = g(f(h(x)))$

$= g(f(7x-2)) = g((7x-2)^3)$
$= g(343x^3 - 294x^2 + 84x - 8)$
$= 4(343x^3 - 294x^2 + 84x - 8)^2 + 1$

2 Marks

$g \circ f \circ g(x) = g(f(g(x))$

$= g(f(4x^2+1)) = g((4x^2+1)^3)$

$= g(64x^6 + 48x^4 + 12x^2 + 1)$

$= 4(64x^6 + 48x^4 + 12x^2 + 1)^2 + 1$

2 Marks

B) p: I play football, q: I can study
r: I pass DBMS

2 Marks

i) $p \to \sim q$, ii) $p \vee r$, iii) $p \wedge q$

3 Marks

C) $p_1 = P(H) = \frac{1}{2}$, $p_2 = P(T) = \frac{1}{2}$

1 Mark

$p_1' = P(\text{two black balls from urn } A) = \frac{^3C_2}{^8C_2}$

1 Mark

$p_2' = P(\text{2 black balls from urn } B) = \frac{^7C_2}{^8C_2}$

1 Mark

$\therefore$ Required probability

$= \frac{p_1 p_1'}{P_1 P_1' + P_2 P_2'} = \frac{\frac{1}{2} \times \frac{^3C_2}{^8C_2}}{\frac{1}{2} \times \frac{^3C_2}{^8C_2} + \frac{1}{2} \times \frac{^7C_2}{^8C_2}} = \frac{1}{8}$

2 Marks

Question No

D) $(p \to q) \land (\sim p \land q)$

$= (\sim p \lor q) \land (\sim p \land q)$ -- (Implication)

$= \sim p \land (\sim p \land q) \lor q \land (\sim p \land q)$ -- (Distributivity)

$= [(\sim p \land \sim p) \land q] \lor [q \land (q \land \sim p)]$ -- (Associativity and Commutativity)

$= (\sim p \land q) \lor ((q \land q) \land \sim p)$ -- (Idempotent and associativity)

$= (\sim p \land q) \lor (q \land \sim p)$ -- (Idempotent)

$= (\sim p \land q) \lor (\sim p \land q)$ -- (Commutativity)

$= \sim p \land q$ -- (Idempotent)

E) A Eulerian path which is a circuit is called as a Eulerian circuit. 1 Mark

A circuit in a graph G which contains all vertices exactly once is Hamiltonian circuit. 1 Mark

Given graph has a Eulerian circuit

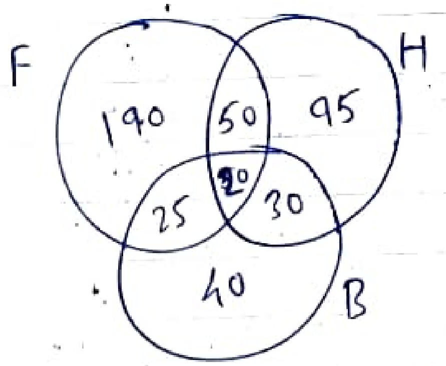
~~f-a-e-b-a-d~~

a-b-d-a-e-d-c-b-e-f-a 2 Marks

Given graph has Hamiltonian circuit

f-a-b-c-d-e-f 1 Mark

F)



$n(F) = 285$, $n(H) = 195$, $n(B) = 115$, $n(F \cap H) = 70$, $n(H \cap B) = 50$, $n(F \cap B) = 45$

2 Marks

$\because$ 50 don't watch any of the games,

$n((F \cup H \cup B)') = 50$

$\therefore n(F \cup H \cup B) = 450$

$\therefore n(F \cap H \cap B) = n(F) + n(H) + n(B) - n(F \cap H) - n(H \cap B) - n(F \cap B) + n(F \cup H \cup B) = 20$ 2 Marks

No. of people watching exactly 1 game $= 190 + 95 + 40$

$= 325$ 1 Mark