QP-2 SE - EXTC Lect^n - III

Q1 A) $L(e^{2t} \sin 3t \cdot \sin 2t)$

$\sin 3t \sin 2t = \frac{1}{2}[\cos 5t - \cos t]$

$L[\sin 3t \sin 2t] = \frac{1}{2}[L(\cos 5t) - L(\cos t)]$

$= \frac{1}{2}\left[\frac{s}{s^2+25} - \frac{s}{s^2+1}\right]$

$L[e^{2t} \sin 3t \cdot \sin 2t] = \frac{1}{2}\left[\frac{s-2}{(s-2)^2+25} - \frac{s-2}{(s-2)^2+1}\right]$

$= \frac{1}{2}\left[\frac{s-2}{s^2-4s+29} - \frac{s-2}{s^2-4s+5}\right]$

$= \frac{1}{2}\left[\frac{s-2}{s^2-4s+5} - \frac{s-2}{s^2-4s+29}\right]$

$= \frac{s-2}{2}\left[\frac{(s^2-4s+29) - (s^2-4s+5)}{(s^2-4s+5)(s^2-4s+29)}\right]$

$= \left(\frac{s-2}{2}\right)\left[\frac{24}{(s^2-4s+5)(s^2-4s+29)}\right]$

$= \frac{12(s-2)}{(s^2-4s+5)(s^2-4s+29)}$

B) Let $f(x) = \sum b_n \sin nx$ $[\because L = \pi]$

$b_n = \frac{2}{\pi}\int_0^{\pi} x^2 \sin nx \, dx$

$= \frac{2}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - (2x)\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$

$= \frac{2}{\pi}\left[\frac{\pi^2}{n}(-\cos n\pi) - 0 + \frac{2}{n^3}\cos n\pi - \frac{2}{n^3}\right]$

$= \frac{2}{\pi}\left[-\frac{\pi^2}{n}(-1)^n + \frac{2}{n^3}(-1)^n - \frac{2}{n^3}\right]$

$= \begin{cases} \frac{2}{\pi}\left[\frac{\pi^2}{n} - \frac{4}{n^3}\right] & n \text{ is odd} \\ \frac{2}{\pi}\left[-\frac{\pi^2}{n}\right] & n \text{ is even} \end{cases}$

$$\therefore x^2 = \frac{2}{\pi}\left[\left(\frac{\pi^2}{1} - \frac{4}{1^3}\right)\sin x - \frac{\pi^2}{2}\sin 2x + \left(\frac{\pi^2}{3} - \frac{4}{3^3}\right)\sin 3x - \cdots\right]$$

(5) $f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$

$f(z) = u + iv$

$u = x^2 + 2axy + by^2$, $v = cx^2 + 2dxy + y^2$

$u_x = 2x + 2ay$, $v_x = 2cx + 2dy$

$u_y = 2ax + 2by$, $v_y = 2dx + 2y$

Since $f(z)$ is analytic, C.R. equations are satisfied

$u_x = v_y$ & $u_y = -v_x$

$2x + 2ay = 2dx + 2y$ & $2ax + 2by = -2cx - 2dy$

$$\boxed{a = 1, b = -1, c = -1, d = 1}$$

D)

X:	23	27	28	29	30	31	33	35	36	39
Y:	18	22	23	24	25	26	28	29	30	32

Let us assume 30 & 25 to be the means of x & y respectively

$$r = \frac{\sum d_x d_y - \frac{(\sum d_x)(\sum d_y)}{N}}{\sqrt{\sum d_x^2 - \frac{(\sum d_x)^2}{N}}\sqrt{\sum d_y^2 - \frac{(\sum d_y)^2}{N}}}$$

$N = 10$, $\bar{X} = 30$, $\bar{Y} = 25$

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Sr.	X	d_x	d_x^2	Y	d_y	d_y^2	$d_x d_y$
1	23	-7	49	18	-7	49	49
2	27	-3	9	22	-3	9	9
3	28	-2	4	23	-2	4	4
4	29	-1	1	24	-1	1	1
5	30	0	0	25	0	0	0
6	31	1	1	26	1	1	1
7	33	3	9	28	3	9	9
8	35	5	25	29	4	16	20
9	36	6	36	30	5	25	30
10	39	9	81	32	7	49	63
N=10		$\Sigma d_x = 11$	$\Sigma d_x^2 = 215$		$\Sigma d_y = 7$	$\Sigma d_y^2 = 163$	186

$$\therefore r = \frac{186 - \frac{11 \times 7}{10}}{\sqrt{215 - \frac{(11)^2}{10}}\sqrt{163 - \frac{(7)^2}{10}}}$$

$$= \frac{186 - 7.7}{\sqrt{215 - 12.1}\sqrt{163 - 4.9}}$$

$$= \frac{178.3}{\sqrt{202.9}\sqrt{158.1}} = \boxed{0.9948}$$

(E) P_1 = Probability he speaks truth $= \frac{3}{5}$

P_1' = Probability of an ace $= \frac{1}{6}$

P_2 = Probability he speaks a lie $= \frac{2}{5}$

P_2' = Probability of not ace $= \frac{5}{6}$.

$\therefore$ P(he speaks truth when ace has occured)

$$= \frac{P_1 P_1'}{P_1 P_1' + P_2 P_2'} = \frac{3}{3+10} = \frac{3}{13}$$

Space for Marks | Question No.

(F) (i) Compute the mean (λ)

$$\lambda = \bar{x} = \frac{\Sigma f x}{\Sigma f} = \frac{100}{200} = 0.5$$

$$\boxed{\lambda = 0.5}$$

ii) Poisson probability formula

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}, \quad \lambda = 0.5$$

$$e^{-0.5} = 0.60653$$

iii) Compute probabilities & expected frequencies

$$E(x) = N \times P(X=x) = 200 \times P(X=x)$$

X	Formula	P(x)	Expected E(x)
0	$e^{-0.5}(0.5)^0/0! = 0.60653$	0.60653	$200 \times 0.6065 = 121.3$
1	$e^{-0.5}(0.5)^1/1! = 0.30327$	0.3033	$200 \times 0.3033 = 60.7$
2	$e^{-0.5}(0.5)^2/2! = 0.07582$	0.0758	$200 \times 0.0758 = 15.2$
3	$e^{-0.5}(0.5)^3/3! = 0.01264$	0.0126	$200 \times 0.0126 = 2.5$
4	$e^{-0.5}(0.5)^4/4! = 0.00158$	0.0016	$200 \times 0.0016 = 0.3$

iv) Comparison

X	Observed f	Expected f
0	122	121.3
1	60	60.7
2	15	15.2
3	2	2.5
4	1	0.3
Total	200	200

$\therefore$ The fitted Poisson distribution is

$$\boxed{P(X=x) = \frac{e^{-0.5}(0.5)^{x}}{x!}}, \quad x = 0, 1, 2 \ldots$$

Q2 A) $\int_0^{\infty} e^{-\sqrt{2}t} \frac{\sin t \sinh t}{t} dt = \frac{\pi}{8}$

$L(\sin t) = \frac{1}{s^2+1}$

$L(e^{-t} \sin t) = \frac{1}{(s+1)^2+1}$

$L[\sin t \sinh t] = \frac{1}{2}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right]$

$L\left[\frac{\sin t \sinh t}{t}\right] = \frac{1}{2}\int_s^{\infty}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right] ds$

$= \frac{1}{2}\left[\tan^{-1}(s-1) - \tan^{-1}(s+1)\right]_s^{\infty}$

$= \frac{1}{2}\left[\left\{\frac{\pi}{2} - \tan^{-1}(s-1)\right\} - \left\{\frac{\pi}{2} - \tan^{-1}(s+1)\right\}\right]$

$= \frac{1}{2}\left[\tan^{-1}(s+1) - \tan^{-1}(s-1)\right]$

Now Let $\tan^{-1}(s+1) = \alpha$ $\quad \tan^{-1}(s-1) = \beta$

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta} = \frac{(s+1) - (s-1)}{1 + (s^2-1)}$

$= \frac{2}{\cancel{\ast}\, s^2}$

$\alpha - \beta = \tan^{-1}\left(\frac{2}{s^2}\right)$

$\therefore \tan^{-1}(s+1) - \tan^{-1}(s-1) = \tan^{-1}\frac{2}{s^2}$

$\therefore L\left(\frac{\sin t \sinh t}{t}\right) = \frac{1}{2}\tan^{-1}\frac{2}{s^2}$

$\int_0^{\infty} e^{-st} \frac{\sin t \sinh t}{t} dt = \frac{1}{2}\tan^{-1}\frac{2}{s^2}$

Put $s = \sqrt{2}$

$\therefore \int_0^{\infty} e^{-\sqrt{2}t}\left(\frac{\sin t \sinh t}{t}\right) dt = \frac{1}{2}\tan^{-1}\frac{2}{2} =$

$= \frac{1}{2} \cdot \frac{\pi}{4} = \frac{\pi}{8}$

Space for Marks | B) Question No.

$$L^{-1}\left[\frac{1}{(s^2+4s+13)^2}\right] = L^{-1}\left[\frac{1}{(s+2)^2+3^2}\right]^2$$

$$= e^{-2t} \cdot L^{-1}\left[\frac{1}{s^2+3^2}\right]^2$$

$L^{-1}\left[\frac{1}{s^2+3^2}\right]^2$ by Convolution Theorem.

Let $\phi_1(s) = \frac{1}{s^2+3^2}$, $\phi_2(s) = \frac{1}{s^2+3^2}$; $\phi(s) = \frac{1}{(s^2+3^2)^2}$

$$L^{-1}\left[\frac{1}{s^2+3^2}\right] = \frac{1}{3}\sin 3t = f(t)$$

$$L^{-1}\left[\frac{1}{s^2+3^2}\right] = \frac{1}{3}\sin 3t = g(t)$$

$$\therefore L^{-1}[\phi(s)] = \frac{1}{9}\int_0^t \sin 3u \cdot \sin 3(t-u)\,du$$

$$= -\frac{1}{18}\int_0^t [\cos 3t - \cos(6u-3t)]\,du$$

$$= -\frac{1}{18}\left[u\cos 3t - \frac{\sin(6u-3t)}{6}\right]_0^t$$

$$= -\frac{1}{18}\left[t\cos 3t - \frac{\sin 3t}{6} - \frac{\sin 3t}{6}\right]$$

$$= \frac{1}{18}\left[\frac{\sin 3t}{3} - t\cos 3t\right]$$

$$\therefore \boxed{L^{-1}\left[\frac{1}{(s^2+4s+13)^2}\right] = \frac{e^{-2t}}{18}\left(\frac{\sin 3t}{3} - t\cos 3t\right)}$$

(c) Let $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ — ①

$$\therefore a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi}\int_0^{2\pi} \frac{(\pi-x)^2}{4}\,dx$$

$$= \frac{1}{8\pi}\left[\frac{(\pi-x)^3}{-3}\right]_0^{2\pi} = -\frac{1}{24\pi}\left[-\pi^3-\pi^3\right]$$

$$\boxed{a_0 = \frac{\pi^2}{12}}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi} \frac{(\pi-x)^2}{4}\cos nx\,dx$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right) - 2(\pi-x)(-1)\left(-\frac{\cos nx}{n^2}\right) + 2(-1)(-1)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\left\{0 + \frac{2\pi\cos 2n\pi}{n^2} + 0\right\} - \left\{0 - \frac{2\pi}{n^2} - 0\right\}\right]$$

$$= \frac{1}{4\pi}\left[\frac{2\pi}{n^2} + \frac{2\pi}{n^2}\right] = \frac{1}{n^2}$$

$$\boxed{a_n = \frac{1}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{(\pi-x)^2}{4}\sin nx\,dx$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(-\frac{\cos nx}{n}\right) - 2(\pi-x)(-1)\left(-\frac{\sin nx}{n^2}\right) + 2(-1)(-1)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\left(-\frac{\pi^2\cos 2n\pi}{n} + 0 + \frac{2\cos 2n\pi}{n^3}\right) - \left(-\frac{\pi^2}{n} + 0 + \frac{2}{n^3}\right)\right]$$

$$= \frac{1}{4\pi}\left[-\frac{\pi^2}{n} + \frac{2}{n^3} + \frac{\pi^2}{n} - \frac{2}{n^3}\right] = 0$$

$$\boxed{b_n = 0}$$

$$\therefore \boxed{\left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2}\cos nx}$$

$$\left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \frac{1}{1^2}\cos x + \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x + \text{——}$$

Put $x = 0$

$$\frac{\pi^2}{4} = \frac{\pi^2}{12} + \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \text{——}$$

$$\boxed{\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \text{——}}$$

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D)

$$u = 3x^2y + 2x^2 - y^3 - 2y^2$$

$U_x = 6xy + 4x$, $U_y = 3x^2 - 3y^2 - 4y$

$U_{xx} = 6y + 4$, $U_{yy} = -6y - 4$

$\therefore U_{xx} + U_{yy} = (6y+4) + (-6y-4) = 0$

$\therefore$ U is Harmonic function.

Since $u = 3x^2y + 2x^2 - y^3 - 2y^2$

By milne Thom. method

$U_x = \phi_1 = 6xy + 4x$, $U_y = \phi_2 = 3x^2 - 3y^2 - 4y$

$\therefore f'(z) = \phi_1(z,0) - i\,\phi_2(z,0)$

$= 4z - i\,3z^2$

$f'(z) = 4z - i\,3z^2$

$\therefore \boxed{f(z) = 2z^2 - i \cdot z^3 + c}$ is the Required Analytic function

$f(z) = 2(x+iy)^2 - i\,(x+iy)^3$

$= 2(x^2 + 2ixy - y^2) - i(x^3 + 3ix^2y - 3xy^2 - iy^3)$

$= (2x^2 - 2y^2 + 3x^2y - y^3) + i(4xy - x^3 - 3xy^2)$

$\therefore \boxed{v = 4xy - x^3 - 3xy^2}$ Harmonic Conjugate

E) (i) $f(x) = A + Bx$ $\quad 0 \le x \le 1$

Since $f(x)$ is a probability distribution

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^1 (A + Bx)\,dx = 1$, $\therefore \left[Ax + \frac{Bx^2}{2}\right]_0^1 = 1$

$A + \frac{B}{2} = 1$

Since the mean is $\frac{1}{3}$,

$\therefore \int_0^1 x f(x)\,dx = \frac{1}{3} \quad \therefore \int_0^1 (A+Bx)x\,dx = \frac{1}{3}$

$\therefore \left[\frac{Ax^2}{2} + \frac{Bx^3}{3} \right]_0^1 = \frac{1}{3} \Rightarrow \frac{A}{2} + \frac{B}{3} = \frac{1}{3}$

$\therefore 3A + 2B = 2 \longrightarrow (ii)$

solving (i) & (ii) $\boxed{A = 2,\ B = -2}$

$\therefore$ The pdf is $\boxed{f(x) = 2 - 2x.}$ $\quad 0 \le x \le 1$

(ii) Since $\Sigma P_i = 1$

$\frac{K-6}{5} + \frac{2}{K} + \frac{14}{5K} = 1 \Rightarrow K^2 - 11K + 14 = 0$
$(K-8)(K-3) = 0$

$K = 8$ or 3

But when $K=3$, $P(X=0) = \frac{3-6}{5} = -\frac{3}{5}$ which is impossible.

$\therefore \boxed{K = 8}$

$\therefore$ The pdf is

X	0	10	15
P(x)	2/5	1/4	7/20

$\therefore E(X) = \Sigma P_i X_i = \frac{5}{2} + \frac{21}{4} = \frac{31}{4}$.

(F) We have S.N.V. $Z = \frac{X - m}{\sigma} = \frac{X - 520}{60}$

(i) Number with incomes betn 400 & 550

$Z_1 = \frac{400-520}{60} = -2.00, \quad Z_2 = \frac{550-520}{60} = 0.50$

From standard normal table

$\phi(0.50) = 0.6915, \ \phi(-2.00) = 0.0228$

So $P(400 < X < 550) = \phi(0.50) - \phi(-2.00)$
$= 0.6915 - 0.0228$
$= 0.6687$.

No. of persons $= 0.6687 \times 10000 = 6687$.

(ii) Lowest income of the richest 500
Richest 500 out of 10000 = top 5%
The cutoff is the 95 percentile.
find x with $P(X < x) = 0.95$.
$Z_{0.95} = 1.645$. Thus

$x = \mu + z\sigma$
$= 520 + 1.645 \times 60$
$= 520 + 98.7$
$\boxed{x = 618.7}$ $\approx$ Rs 619

Q3 A) $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right]$

Let $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right] = \frac{A}{s-1} + \frac{Bs+C}{s^2+2s+5}$

$5s+3 = A(s^2+2s+5) + (s-1)(Bs+c)$
$5s+3 = A \cdot s^2 + 2As + 5A + Bs^2 + Cs - Bs - C$
$A + B = 0$ $\quad B = -A$
$2A - B + C = 5 \Rightarrow 3A + C = 5$
$5A - C = 3 \Rightarrow 5A - C = 3$
$8A = 8$
$\boxed{A = 1}$
$\boxed{B = -1}$
$\therefore C = 5A - 3$
$\boxed{C = 2}$

$\therefore L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right] = L^{-1}\left[\frac{1}{s-1}\right] + L^{-1}\left[\frac{-s+2}{s^2+2s+5}\right]$

$= e^{t} - L^{-1}\left[\frac{s}{(s+1)^2+4}\right] + 2L^{-1}\left[\frac{1}{(s+1)^2+4}\right]$

$= e^{t} - e^{-t}\cos 2t + e^{-t}\sin 2t$

(B) $f(x) = 4-x^2 \quad (0,2), \; 2L=2, \; L=1$

$f(x) = a_0 + \Sigma a_n \cos\left(\frac{n\pi x}{L}\right) + \Sigma b_n \sin\left(\frac{n\pi x}{L}\right)$

$f(x) = a_0 + \Sigma a_n \cos n\pi x + \Sigma b_n \sin n\pi x$ — (1)

$a_0 = \frac{1}{2L}\int_0^{2L} (4-x^2)\,dx = \frac{1}{2}\left[4x - \frac{x^3}{3}\right]_0^2$

$= \frac{1}{2}\left[8 - \frac{8}{3}\right] = \frac{8}{3} \qquad \boxed{a_0 = \frac{8}{3}}$

$a_n = \frac{1}{L}\int_0^{2L} (4-x^2) \cos \frac{n\pi x}{L}\,dx$

$= \int_0^2 (4-x^2)\cos n\pi x\,dx$

$= \left[(4-x^2)\frac{\sin n\pi x}{n\pi} - (-2x)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) + (-2)\left(-\frac{\sin n\pi x}{n^3\pi^3}\right)\right]_0^2$

$= \left[\left(0 - \frac{4}{n^2\pi^2} + 0\right) - (0)\right] = \frac{-4}{n^2\pi^2}$

$\boxed{a_n = \frac{-4}{n^2\pi^2}}$

$b_n = \frac{1}{L}\int_0^{2L} (4-x^2) \sin \frac{n\pi x}{L}\,dx$

$= \int_0^2 (4-x^2)\sin n\pi x\,dx$

$= \left[(4-x^2)\left(-\frac{\cos n\pi x}{n\pi}\right) - (-2x)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right) + (-2)\left(\frac{\cos n\pi x}{n^3\pi^3}\right)\right]_0^2$

$= \left[\left(0 - 0 - \frac{2}{n^3\pi^3}\right) - \left(-\frac{4}{n\pi} - \frac{2}{n^3\pi^3}\right)\right] = \frac{4}{n\pi}$

$\boxed{b_n = \frac{4}{n\pi}}$

$\therefore f(x) = \frac{8}{3} - \frac{4}{\pi^2}\Sigma\frac{\cos n\pi x}{n^2} + \frac{4}{\pi}\Sigma\frac{\sin n\pi x}{n}$

$4-x^2 = \frac{8}{3} - \frac{4}{\pi^2}\left[\frac{1}{1^2}\cos \pi x + \frac{1}{2^2}\cos 2\pi x + \frac{1}{3^2}\cos 3\pi x + \cdots\right]$

$+ \frac{4}{\pi}\left[\frac{1}{1}\sin \pi x + \frac{1}{2}\sin 2\pi x + \frac{1}{3}\sin 3\pi x \cdots\right]$

(c)

$u+v = e^x(\cos y+\sin y)$

$f(z) = u+iv \qquad \therefore if(z) = iu - v$

$(1+i)f(z) = (u-v) + i(u+v) = U + iV$

$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(u+v) = e^x(\cos y + \sin y) = \psi_1(x,y)$

$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(u+v) = e^x(-\sin y + \cos y) = \psi_2(x,y)$

By milne-Thompson method

$(1+i)f'(z) = \frac{\partial V}{\partial y} + i\frac{\partial V}{\partial x} = \psi_2(z,0) + i\psi_1(z,0)$

$= e^z(0+1) + ie^z(1+0) = (1+i)e^z$

$f'(z) = e^z$

$\therefore \boxed{f(z) = e^z + c.}$

D).

X	R_1	Y	R_2	$D^2=(R_1-R_2)^2$
32	3	40	5	4.00
55	9	30	3.5	30.25
49	7.5	70	9	2.25
60	10	20	1	81.00
43	5.5	30	3.5	4.00
37	4	50	7	9.00
43	5.5	72	10	20.25
49	7.5	60	8	0.25
10	1	45	6	25.00
20	2	25	2	0

$\Sigma D^2 = 176$

$\therefore R = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \frac{1}{12}(m_3^3 - m_3)\right]}{N^3 - N}$

$m_1 = m_2 = m_3 = 2, N = 10, \Sigma D^2 = 176$

$\therefore R = 1 - 1.076 = -0.076$

$\boxed{R = -0.076.}$

Space for Marks | Question

E)

Sr.	X	X^2	Y	Y^2	XY
1	5	25	11	121	55
2	6	36	14	196	84
3	7	49	14	196	98
4	8	64	15	225	120
5	9	81	12	144	108
6	10	100	17	289	170
7	11	121	16	256	176
N=7	56	476	99	1427	811

Now, the line of regression of y on X is

$$\boxed{Y = A + BX}$$

The Normal equations are

$\Sigma Y = NA + B\Sigma X$ $\qquad 99 = 7A + 56B$

$\Sigma XY = A\Sigma X + B\Sigma X^2$ $\qquad 811 = 56A + 476B$

Solving

$A = 8.7143$

$B = 0.6789$

$\therefore$ Equation of the line of regression of Y on X is

$$\boxed{Y = 8.7143 + 0.6789X}$$

F) We have, here x taking values 1, 2, 3, 4, 5, 6 each with probability $1/6$

$M_0(t) = E(e^{tx_i}) = \Sigma p_i e^{tx_i}$

$= \frac{1}{6}e^t + \frac{1}{6}e^{2t} + \frac{1}{6}e^{3t} + \longrightarrow + \frac{1}{6}e^{6t}$

$$\boxed{M_0(t) = \frac{1}{6}(e^t + e^{2t} + \longrightarrow + e^{6t})}$$ MGF.

$\therefore \mu_1' = \left[\frac{d}{dt} M_0(t)\right]_{t=0} = \frac{1}{6}\left[e^t + 2e^{2t} + 3e^{3t} + \longrightarrow + 6e^{6t}\right]_{t=0}$

$= \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6]$

$\mu_1' = \frac{21}{6}$ $\qquad \boxed{\mu_1' = \frac{7}{2}}$ $\rightarrow$ Mean

Space for Marks	Question No.

$$\mu_2' = \left[\frac{d^2}{dt^2} M_0(t)\right]_{t=0}$$

$$= \frac{1}{6}\left[e^t + 4e^{2t} + 9e^{3t} + \text{———} + 36e^{6t}\right]_{t=0}$$

$$= \frac{1}{6}\left[1 + 4 + 9 + \text{———} + 36\right]$$

$$\mu_2' = \frac{91}{6}$$

$$\therefore \mu_2 = \mu_2' - (\mu_1')^2 = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$$

$\therefore$ Mean $= \frac{7}{2}$, Variance $= \frac{35}{12}$.