Q.1. a) (1)

$f(t) = \cos 3t \cos 2t \cos t$

$= \frac{1}{2}(\cos 5t + \cos t)\cos t$ ——— 1

$f(t) = \frac{1}{4}(\cos 6t + \cos 4t + \cos 2t + 1)$ ——— 3

$L[f(t)] = \frac{1}{4}\left[\frac{s}{s^2+36} + \frac{s}{s^2+16} + \frac{s}{s^2+4} + \frac{1}{s}\right]$ ——— 5

b) $L^{-1}[\phi(s)] = -\frac{1}{t} L^{-1}[\phi'(s)]$ ——— (1)

$L^{-1}\left[\log\left(\frac{s+3}{s+4}\right)\right] = -\frac{1}{t} L^{-1}\left[\frac{1}{s+3} - \frac{1}{s+4}\right] = -\frac{1}{t}(e^{-3t} - e^{-4t})$ ——— (3)

$L^{-1}\left[\frac{1}{s}\right] = 1$ ——— (4)

$L^{-1}\left[\frac{1}{s}\log\left(\frac{s+3}{s+4}\right)\right] = \int_0^t \frac{e^{-4u} - e^{-3u}}{u}\, du$ ——— 5

c) $L(\sin u) = \frac{1}{s^2+1}$ (1)

$L\left(\frac{\sin u}{u}\right) = \int_s^\infty \frac{1}{s^2+1}\, ds = \tan^{-1}\frac{\pi}{2} - \tan^{-1} s$

$= \frac{\pi}{2} - \tan^{-1} s. = \cot^{-1} s.$ ——— 3

$L\left[\int_0^t e^{-u}\cdot\frac{\sin u}{u}\, du\right] = \frac{1}{s}\cdot\cot^{-1}(s+1).$ ——— 5

d) $f(x) = x^2$ in $(-\pi, \pi)$.

$\Rightarrow f(x) = x^2$ is an even function. Hence, $b_n = 0$.

$a_0 = \frac{2}{\pi}\int_0^\pi x^2\, dx = \frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^\pi = \frac{2\pi^2}{3}$

$a_n = \frac{2}{\pi}\int_0^\pi x^2 \cos nx\, dx = \frac{4}{n^2}(-1)^n$ $\therefore f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4\cdot(-1)^n}{n^2}\cos nx$

e) Given, $m = 1.5$ ——— (2)

$P(x=n) = \frac{e^{-m}\cdot m^x}{x!} = \frac{e^{-1.5}(1.5)^x}{x!}$ $x = 0, 1, 2, \ldots$

i) $P(x=0) = 0.2231$. (no-demand) ——— (3)

ii) $P(x \geq 2) = 1 - [P(x=0) + P(x=1) + P(x=2)] = 1 - (0.2231 + 0.3347 + 0.2510)$

$P(x \geq 2) = 0.1912$ — some demand is refused. ——— (5).

f) SNV. $z = \frac{x-m}{\sigma} = \frac{x-70}{5}$ ——— (1)

When $x = 60$, $z = -2$, When $x = 75$, $z = 1$. ——— 3

$P(60 \leq x \leq 75) = P(-2 < z < 1) =$ Area from $(z=0$ to $z=2)$ + Area from $(z=0$ to $z=1)$

$= 0.4772 + 0.3413 = 0.8185$ ——— 4

No. of students = 819 ——— 5

Q.2. A) Prove that $\int_0^\infty \frac{\sin 2t + \sin 3t}{t e^t}\,dt = \frac{3\pi}{4}$

$\rightarrow L(\sin 2t + \sin 3t) = \frac{2}{s^2+4} + \frac{3}{s^2+9}$ — 2

$L\left(\frac{\sin 2t + \sin 3t}{t}\right) = \left(\tan^{-1}\frac{s}{2} + \tan^{-1}\frac{s}{3}\right)_s^\infty = \pi - \left[\tan^{-1}\left(\frac{s}{2}\right) + \tan^{-1}\left(\frac{s}{3}\right)\right]$ — 4

$= \pi - \tan^{-1}\left(\frac{\frac{s}{2}+\frac{s}{3}}{1-\frac{s}{2}\cdot\frac{s}{3}}\right) = \pi - \tan^{-1}\left(\frac{6s}{6-s^2}\right)$ — 6, 7

put $s=1$. — 8

$\int_0^\infty \frac{\sin 2t + \sin 3t}{t e^t}\,dt = \pi - \tan^{-1}(1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ — 10

Q.2. B) Find the $L^{-1}\left[\frac{(s+2)^2}{(s^2+4s+8)^2}\right]$ by using convolution theorem.

$f(s) = \frac{(s+2)^2}{[(s+2)^2+4]^2} = e^{-2t} L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right]$ — 3

$L^{-1}\left[\frac{s^2}{(s^2+2)^2}\right] = \int_0^t \cos 2u \cdot \cos 2(t-u)\,du$. — 5

$= \frac{1}{2}\int_0^t [\cos 2t + \cos(4u-2t)]\,du$ — 6

$= \frac{1}{2}\left[u\cos 2t + \frac{1}{4}\sin(4u-2t)\right]_0^t$ — 7

$= \frac{1}{2}\left[t\cos 2t + \frac{1}{2}\sin 2t\right]$ — 8

$\therefore L^{-1}[f(s)] = e^{-2t}\cdot\frac{1}{4}(2t\cos 2t + \sin 2t)$ — 10

Q.3. A) $f(x) = \sqrt{1-\cos x} = \sqrt{2}\cdot \sin\frac{x}{2}$ ——— (1)

$a_0 = \frac{1}{\pi}\int_0^{\pi} \sqrt{2}\sin\frac{x}{2}\,dx = 4\frac{\sqrt{2}}{\pi}$ ——— (4)

$a_n = \frac{1}{\pi}\int_0^{2\pi} \sqrt{2}\sin\frac{x}{2}\cdot\cos nx\,dx = \frac{-4\sqrt{2}}{\pi(4n^2-1)}$ ——— (7)

$b_n = \frac{1}{\pi}\int_0^{2\pi} \sqrt{2}\sin\frac{x}{2}\cdot\sin nx\,dx = 0$ ——— (8)

$\therefore f(x) = \frac{2\sqrt{2}}{\pi} + -\frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cdot\cos nx$ ——— (9)

put $x = 0$.

$\frac{1}{2} = \sum_{n=1}^{\infty}\frac{1}{4n^2-1}$ ——— (10)

Q.3. B) $f(x) = 4-x^2$ in $(0,2)$. $\boxed{l=1}$ ——— (1)

$a_0 = \frac{1}{l}\int_0^{2l}(4-x^2)\,dx = \frac{1}{1}\int_0^{2}(4-x^2)\,dx = \left(4x-\frac{x^3}{3}\right)_0^2 = 8-\frac{8}{3} = 8\left(\frac{2}{3}\right) = \frac{16}{3}$ ——— (3)

$a_n = \frac{1}{1}\int_0^{2}(4-x^2)\cos(n\pi x)\,dx = -\frac{4}{n^2\pi^2}$ ——— (6)

$b_n = \frac{1}{1}\int_0^{2}(4-x^2)\sin(n\pi x)\,dx = \frac{4}{n\pi}$ ——— (8)

$f(x) = 4-x^2 = \frac{8}{3} + \sum_{n=1}^{\infty}\left(\frac{-4}{n^2\pi^2}\right)\cos n\pi x + \sum_{n=1}^{\infty}\frac{4}{n\pi}\sin n\pi x$ ——— (10)

Q.4. A) $V_{xx} + V_{yy} = 0$ ——— (4) ——— $\boxed{5}$

$f(z) = \int[\psi_1(z,0) + i\,\psi_2(z,0)]\,dz + c = \int e^z(z+1)\,dz$ ——— $\boxed{6}$

$= (z+1)e^z + -(1)e^z + c$

$f(z) = ze^z + c$ ——— $\boxed{7}$

$f(z) = (x+iy)e^{x+iy} + c$ ——— $\boxed{8}$

$\therefore v = e^x(x\cos y - y\sin y)$ ——— $\boxed{10}$ is harmonic conjugate.

Q.4. B) Let $u = x^3y - xy^3 - c$. —(1) (4)

$u_x = 3x^2y - y^3 = \phi_1(x,y)$, —(2) $u_y = x^3 - 3xy^2 = \phi_2(x,y)$ —(3)

$\phi_1(z,0) = 0$, — $\phi_2(z,0) = z^3$. —[4]

$\therefore f(z) = \int (0 - iz^3)\,dz + c = -i\frac{z^4}{4} + c$. —[7]

$f(z) = -i\frac{(x+iy)^4}{4} + c$ —[8]

$= -\frac{i}{4}(x^4 + 4x^3iy + 6x^2i^2y^2 + 4xi^3y^3 + i^4y^4) + c$

$\therefore \phi = -\frac{1}{4}(x^4 - 6x^2y^2 + y^4)$ —[10]

Q.5. A) $\bar{x} = 68$, $\bar{y} = 69$. $x = x - 68$, $y = y - 69$. —(2)

x	$x = x-68$	x^2	y	$y = y-69$	y^2	xy
65	-3	9	67	-2	4	6
66	-2	4	68	-1	1	2
67	-1	1	65	-4	16	4
67	-1	1	68	-1	1	1
68	0	0	72	3	9	0
69	1	1	72	3	9	3
70	2	4	69	0	0	0
72	4	16	71	2	4	8
544		36	552		44	24

—(6)

—(7)

$N = 8$, $r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} = \frac{24}{\sqrt{36 \times 44}} = \frac{24}{8\sqrt{44}} = \frac{4}{\sqrt{44}} = 0.6030$

—(10)

Q.5. B) Let $x + 3y = 5$ be the line of regression of x on y. —(1)

$\therefore b_{xy} = -3$ —(2)

Let $4x + 3y = 8$ " " " —(3) " of y on x.

$b_{yx} = -\frac{4}{3}$. —(4) $\therefore r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{-\frac{4}{3} \times -3} = \sqrt{4} = 2 > 1$. —(5)

Hence, our assumptions are wrong.

Now, let $x + 3y = 5$ be the line of regression of y on x. —(6)

$= -\frac{1}{3}$ —(7) & $b_{xy} = -\frac{3}{4}$. —(8)

$r = \sqrt{-\frac{1}{3} \times -\frac{3}{4}} = \sqrt{\frac{1}{4}} = 0.5$ —(9)

Hence, the statement is true. —(10)

Q.6. A). To find K: (5)

$$\int_0^1 f(x)\,dx = 1 \quad \text{——(1)}$$

$$\int_0^1 K(x-x^2)\,dx = 1 \quad \text{——(2)}$$

$$\Rightarrow K\left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = 1 \quad \text{——(3)}$$

$$K\left(\frac{1}{2} - \frac{1}{3}\right) = 1 \Rightarrow K\left(\frac{3-2}{6}\right) = 1$$

$$\frac{K}{6} = 1 \quad \therefore K = 6 \quad \text{——(4)}$$

To find mean:

$$E(x) = \int_0^1 x \cdot f(x)\,dx = 6\int_0^1 x(x-x^2)\,dx$$

$$E(x) = \frac{1}{2} = \mu_1' \quad \text{——(6)}$$

To find $E(x^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 6 \cdot x^2(x-x^2)\,dx$

$$\mu_2' = 6\int_0^1 (x^3 - x^4)\,dx = \frac{3}{10}. \quad \text{——(9)}$$

$$\text{Var}(x) = \mu_2' - \mu_1'^2 = \frac{3}{10} - \left(\frac{1}{2}\right)^2 = \frac{3}{10} - \frac{1}{4} = \frac{12-10}{40}$$

$$= \frac{2}{40} = \frac{1}{20} \quad \text{——(10)}$$

Q.6. B)

i) $M_x(t) = E(e^{tx}) = \sum_x e^{tx}\, p(x=x)$ ——(1)

$$M_x(t) = e^{t(0)}\left(\frac{1}{6}\right) + e^{t}\left(\frac{1}{3}\right) + e^{2t}\left(\frac{1}{3}\right) + e^{3t}\left(\frac{1}{6}\right) \quad \text{——(2)}$$

$$M_x(t) = \frac{1}{6} + \frac{1}{3}e^t + \frac{1}{3}e^{2t} + \frac{1}{6}e^{3t} \quad \text{——(3)}$$

ii) First Four Raw Moments $(\mu_1', \mu_2', \mu_3', \mu_4')$.

$$\mu_r' = E(x^r) = \sum x^r \cdot p(x=x) \quad \text{——(4)}$$

$$\mu_1' = \frac{3}{2}, \quad \mu_2' = \frac{19}{6}, \quad \mu_3' = \frac{15}{2}, \quad \mu_4' = \frac{115}{6}. \quad \text{——(6)}$$

iii) First Four central Moments. $(\mu_1, \mu_2, \mu_3, \mu_4)$.

(7)

$$\mu_1 = 0, \quad \mu_2 = \mu_2' - \mu_1'^2 = \frac{19}{6} - \left(\frac{3}{2}\right)^2 = \frac{19}{6} - \frac{9}{4} = \frac{11}{12} \quad \text{——(8)}$$

$$\mu_3 = 0, \quad \mu_4 = \frac{83}{48} = 1.729 \quad \text{——(10)}$$