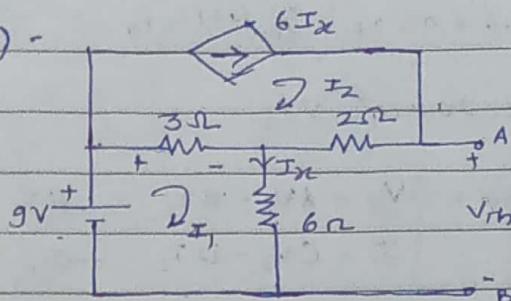


Branch: EXTC Course: Network Theory  
Year/Sem: 3E/III. Course code: ETC-304 Marks: 80

Q1) (a) -

Calculation of  $V_{th}$ :

$$\text{from fig. } I_{in} = I_1 \quad \dots \quad (1)$$

Applying KVL to Mesh 1:

$$9 - 3(I_1 - I_2) - 6I_1 = 0$$

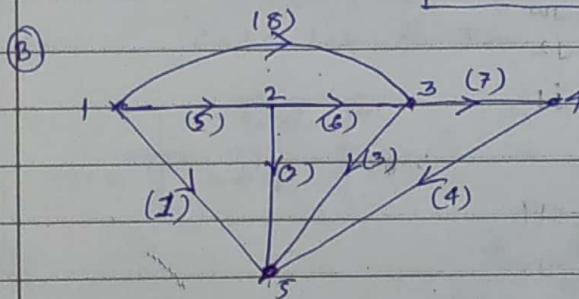
$$9I_1 - 3I_2 = 9 \quad \dots \quad (2)$$

$$\text{from Mesh 2: } I_2 = 6I_x = 6I_1 \quad \therefore 6I_1 - I_2 = 0 \quad \dots \quad (3)$$

$$\text{Solving (2) & (3): } I_1 = -1A \quad I_2 = -6A$$

$$\text{Writing } V_{th} \text{ eqn: } 9 - 3(I_1 - I_2) + 2I_2 - V_{th} = 0$$

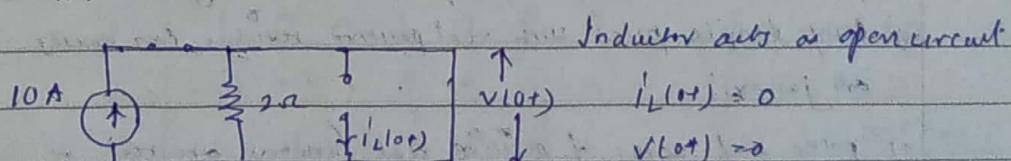
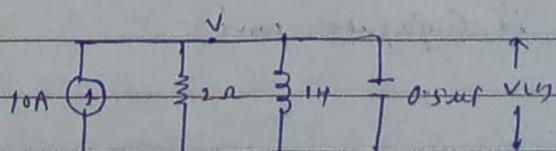
$$V_{th} = -18V$$



(c)

At  $t=0^-$  no current flows through the inductor and there is no voltage across capacitor.

$$I(L(0^-)) = 0 \quad V(C(0^-)) = 0$$

At  $t=0^+$ ,For  $t > 0$ Writing KCL eqn for  $t > 0$ 

$$\frac{V}{2} + \frac{1}{L} \int V dt + 0.5 \times 10^6 \frac{dV}{dt} - 10 = 0$$

$$\text{At } t=0^+ \quad \frac{V(0^+)}{2} + 0 + 0.5 \times 10 \xrightarrow{t} \frac{dV}{dt}(0^+) = 10$$

$$\frac{dV(0^+)}{dt} = 20 \times 10^6 \text{ V/s}$$

d) ABCD parameters are:  $V_1 = AV_2 - BI_2 \quad \text{--- (1)}$

Rewriting eqn (2)  $I_1 = CV_2 - DI_2 \quad \text{--- (2)}$

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2 \quad \text{--- (3)}$$

4)  $V_1 = AV_2 - BI_2$

$$= AV_2 - B \left[ -\frac{1}{D} I_1 + \frac{C}{D} V_2 \right]$$

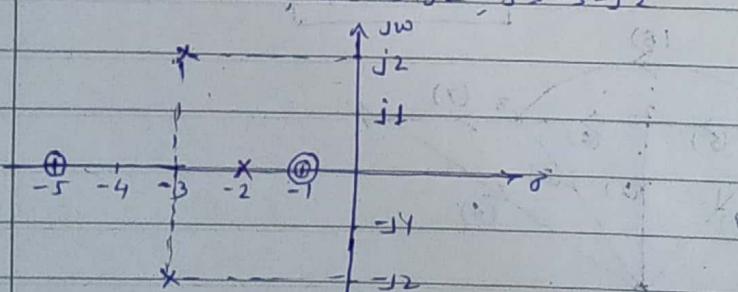
$$V_1 = \frac{B}{D} I_1 + \left[ \frac{AD - BC}{D} \right] V_2 \quad \text{--- (4)}$$

Comparing eqn (3) & (4) with std. h parameters

$$h_{11} = \frac{B}{D}, \quad h_{12} = \frac{AD - BC}{D}, \quad h_{21} = -\frac{1}{D}, \quad h_{22} = \frac{C}{D}$$

d) Function  $F(s)$  has zero at  $s = -1 \pm j$ ,  $s = -5$  and

poles at  $s = -2, s = -3 + j2, s = -3 - j2$



f) properties of positive real function

i) If  $F(s)$  is positive real then  $\frac{1}{F(s)}$  is also positive real

ii) sum of two positive real functions is positive real

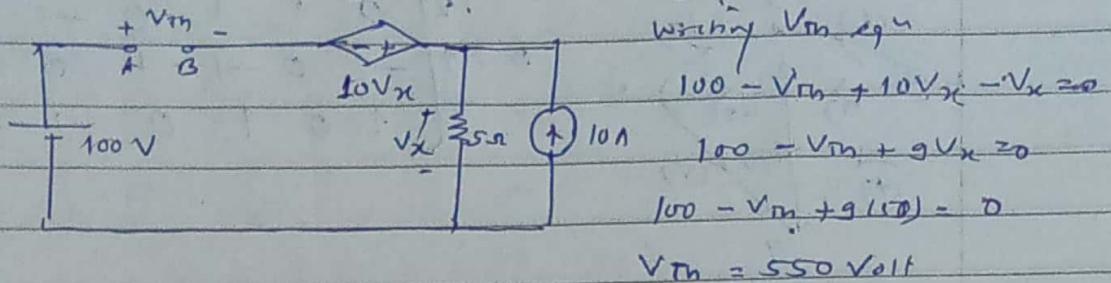
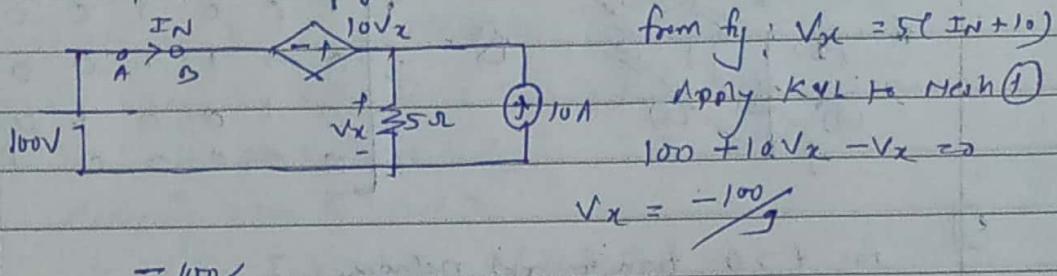
iii) poles & zeros of positive real function cannot have

positive real parts as they cannot be in right half of s-plane

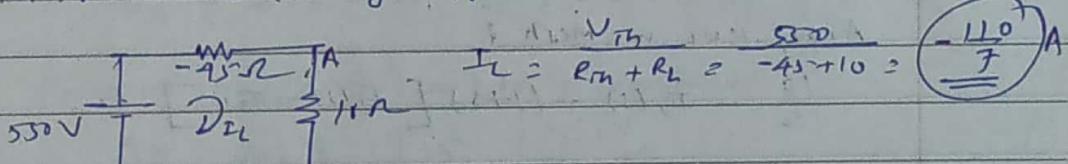
iv) Only simple poles with real positive residue can exist on jw-axis.

v) poles and zeros of positive real function are real or occur in conjugate pairs

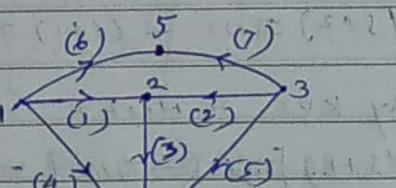
Q2A

Step 1: Calculation of  $V_{Th}$  from fig.  $V_x = 10 \times 5 = 50 \text{ V}$ Step 2: Calculation of  $I_N$ 

$$I_N = -\frac{150}{45} \text{ A}$$

Step III: Calculation of  $R_{Th}$   $R_{Th} = \frac{V_{Th}}{I_N} = \frac{550}{-\frac{150}{45}} = -15 \Omega$ Step IV: Calculation of  $T_N$ 

oriented graph



Incidence matrix (A)

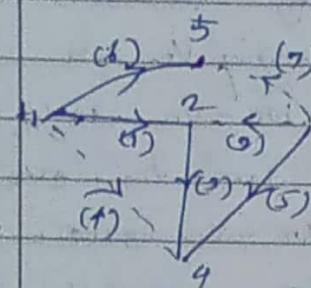
Nodes  $\rightarrow 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7$  Branches  $\rightarrow$ 

1	1	0	0	1	0	1	0	$1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0$
2	-1	-1	1	0	0	0	0	$-1 \ -1 \ 1 \ 0 \ 0 \ 0 \ 0$
3	0	1	0	0	1	0	1	$0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1$
4	0	0	-1	-1	-1	0	0	$0 \ 0 \ -1 \ -1 \ -1 \ 0 \ 0$
5	0	0	0	0	0	-1	-1	$0 \ 0 \ 0 \ 0 \ 0 \ -1 \ -1$

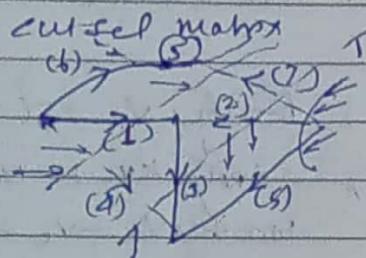
Reduced incidence matrix (Aa)  $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ while eliminating last row,  $A = \begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 \end{bmatrix}$ 

from matrix (Aa)

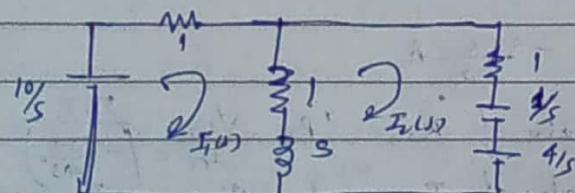
Triangulated matrix (B)



$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & -1 & 1 \end{bmatrix}$$



$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

e) for  $t > 0$  transformed network

Apply KVL to Mesh 1

$$\frac{10}{s} - I_{1,11} - (s+1) [I_{1,11} - I_{1,12}] \Rightarrow$$

$$(s+2) I_{1,12} - (s+1) I_{1,11} = \frac{10}{s} \quad (1)$$

Apply KVL to Mesh 2

$$-(s+1) [I_{2,11} - I_{2,12}] - I_{2,11} - \frac{1}{s} I_{2,12} = \frac{4}{s} \Rightarrow$$

$$-(s+1) I_{2,11} + (s+2 + \frac{1}{s}) I_{2,12} = \frac{4}{s} \quad (2)$$

By cramer's rule

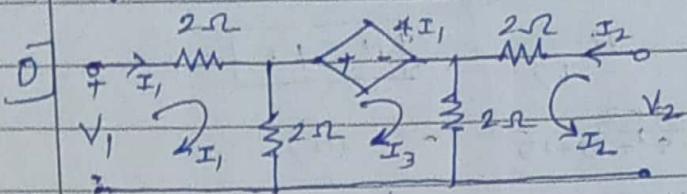
$$I_{1,11} = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} \frac{10}{s} & -(s+1) \\ -4/s & s+2+\frac{1}{s} \end{vmatrix}}{\begin{vmatrix} s+2 & -(s+1) \\ -(s+1) & s+2+\frac{1}{s} \end{vmatrix}} = \frac{3s+5}{s(s+1)}$$

$$\text{By partial fraction: } I_{1,11} = \frac{A_1}{s} + \frac{B_1}{s+1}$$

$$I_{1,11} = \frac{s}{s} - \frac{2}{s+1} \text{ by inverse LT, } I_{1,11} = 5 - 2e^{-t}$$

$$\text{Similarly: } I_{2,11} = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} \frac{10}{s} & 1/s \\ -(s+1) & -4/s \end{vmatrix}}{\begin{vmatrix} s+2 & -1/s \\ -(s+1) & -4/s \end{vmatrix}} = \frac{3}{s+1} - \frac{2}{(s+1)^2}$$

By Inverse LT.  $i_2(t) = 3\bar{e}^t - 2t\bar{e}^t$



Applying KVL to Mesh 1

$$V_1 = 2I_1 + 2(I_1 - I_3) \quad \text{--- (i)}$$

$$V_1 = 4I_1 - 2I_3 \quad \text{--- (i)}$$

Applying KVL to Mesh 2

$$V_2 = 2I_2 + 2(I_2 + I_3) \quad \text{--- (ii)}$$

$$V_2 = 4I_2 + 2I_3 \quad \text{--- (ii)}$$

Applying KVL to Mesh 3

$$-2(I_3 - I_1) - 1I_1 - 2(I_3 + I_2) = 0$$

$$I_1 + I_2 = -2I_3 \quad \text{--- (iii)}$$

put eqn (iii) in (i) & (ii)

$$V_1 = 4I_1 + I_1 + I_2 = 5I_1 + I_2 \quad \text{--- (iv)}$$

$$V_2 = 4I_2 - I_1 - I_2 = -I_1 + 3I_2 \quad \text{--- (v)}$$

Comparing (iv) & (v) with Z parameters

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

To find h-parameters

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (5)(3) - (1)(-1) = 16$$

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{16}{3} \Omega, \quad h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{1}{3}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = \frac{1}{3}, \quad h_{22} = \frac{1}{Z_{22}} = \frac{1}{3} \Omega$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{16}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

E

$$Z(s) = \frac{(R+LS) \frac{1}{C}}{R+LS + \frac{1}{C}} = \frac{\frac{1}{C}(R+LS)}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \quad \text{--- (i)}$$

form pole zero diagram

zero is at  $s = -2$  & poles are at  $s = -1 \pm j\sqrt{3}$

$$Z(s) = H \frac{s+2}{(s+1+j4)(s+1-j4)} = H \frac{s+2}{(s+1)^2 - (j4)^2} = H \frac{s+2}{s^2 + 2s + 17}$$

$$H \text{ d.c.}, \quad 2 = H \cdot \frac{2}{17} \quad \boxed{H = 17}$$

$$Z(s) = 17 \cdot \frac{s+2}{s^2 + 2s + 17} \quad \dots \quad (i)$$

Company eq w.r.t.  $\omega$  with  $c_j$   $\frac{1}{c} = 17$ ,  $\frac{R}{L} = 2$ ,  $\frac{L}{C} = 17$

$$\therefore C = \frac{1}{17} R, \quad L = 17, \quad R = 2 \Omega$$

F Lower I form:

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)} = \frac{2s^2 + 8s + 6}{s^2 + 8s + 12}$$

By Continued fraction expansion

$$\begin{array}{r} s^2 + 8s + 12 \\ \hline 2s^2 + 8s + 6 \quad (s \leftarrow s) \\ -2s^2 - 16s - 24 \\ \hline -8s + 18 \end{array}$$

$\therefore$  negative result, Continued fraction expansion of  $Y(s)$

Cancel out

$$Y(s) = \frac{s^2 + 8s + 12}{2s^2 + 8s + 6}$$

By Continued fraction expansion

$$\begin{array}{r} s^2 + 8s + 6 \\ \hline 2s^2 + 8s + 12 \quad (s \leftarrow \frac{1}{2}s) \\ -2s^2 - 8s - 12 \\ \hline 16s + 9 \\ \hline 2s^2 + 9s \\ \hline 7s^2 + 6s \\ \hline 4s + 9 \quad (\frac{8}{7} \leftarrow y) \\ \hline 4s + 9 \\ \hline -17 \\ \hline \frac{1}{7} \quad (\frac{7}{2}s + 6 \leftarrow \frac{49}{30} \leftarrow z) \\ \hline \frac{2}{7}s \\ \hline 9 \\ \hline 17 \\ \hline 18 \end{array}$$

$$\begin{array}{c} \frac{1}{2}H \\ \hline m \quad m \quad m \\ \hline 2\Omega \quad \frac{7}{8}\Omega \quad \frac{12}{15}\Omega \end{array}$$

Lower I form

Lower II form

Arranging Numerator & Denominator polynomials of  $s$  in ascending order of  $s$ .

Q2 (F) ~~cauver II Ans~~

$$Z(s) = \frac{6 + 8s + 2s^2}{12 + 8s + 8s^2}$$

By Continued fraction expansion

$$12 + 8s + 8s^2 \quad (1 + 8s + 2s^2 \leftarrow 2)$$

$$1 + 8s + \frac{1}{2}s^2$$

$$1s + \frac{3}{2}s^2 \quad 12 + 8s + 2s^2 \leftarrow \frac{3}{2} \leftarrow 1$$

$$12 + \frac{9}{2}s$$

$$\frac{3}{2}s + s^2 \quad 12 + 8s + 2s^2 \leftarrow \frac{3}{2} \leftarrow 2$$

$$1 + \frac{3}{2}s^2$$

$$s + \frac{3}{2}s^2 \quad \frac{2}{2}s + s^2 \leftarrow \frac{98}{108} \leftarrow 1$$

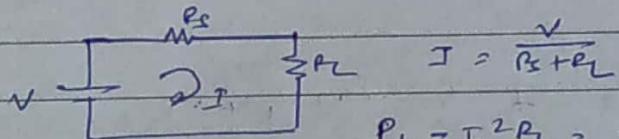
$$\frac{7}{2}s \quad s^2 \quad \frac{5}{4}s^2 \leftarrow \frac{5}{4} \leftarrow 2$$

$$\frac{5}{4}s^2$$

$$1 \leftarrow 0$$

Q3 (A)

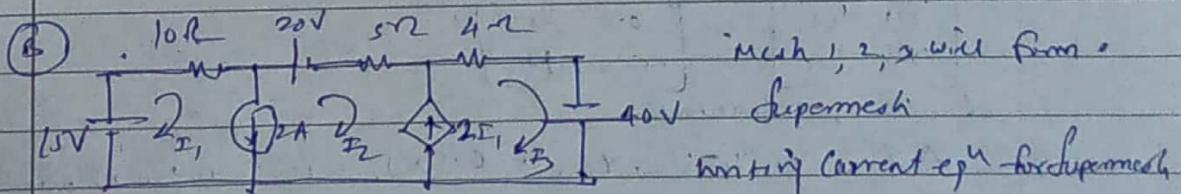
Maximum power Transfer theorem

Maximum power is delivered from source to load when  $R_L = R_S$ 

$$P_L = I^2 R_L = \frac{V^2 R_L}{(R_S + R_L)^2}$$

To determine value of  $R_L$  max power transferred to load.

$$\frac{dP_L}{dR_L} = 0 \quad \frac{dP}{dR_L} = \frac{V^2}{(R_S + R_L)^2} \cdot R_L \quad \therefore R_L = R_S$$



$$I_1 - I_2 = 2 \quad (i)$$

$$I_3 - I_2 = 2I_1 - (ii)$$

$$2I_1 + I_2 - I_3 = 0$$

Applying KVL in outer part of supermesh

$$15 - 10I_1 - 20 - 5I_2 - 4I_3 + 40 = 0$$

$$10I_1 + 5I_2 + 4I_3 = 35 \quad (iii)$$

Solving eqn (i) (ii) &amp; (iii)

$$I_1 = 1.96A, I_2 = -0.04A, I_3 = 3.87A$$

$$I_{W.R} = I_1 = 1.96A$$