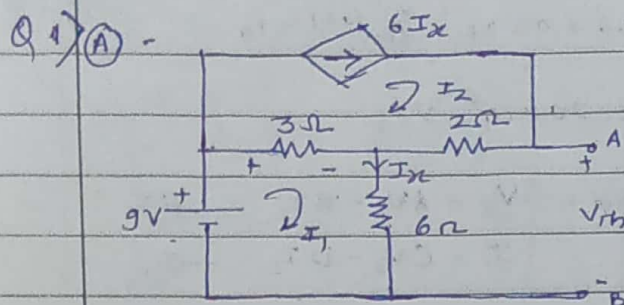


Solution:

Branch: EXTC Course: Network Theory  
 Year/Sem: SE/III Course code: ETC-304 Marks: 80

~~Q1)~~Calculation of  $V_{th}$ From fig,  $I_x = I_1$  - - (1)

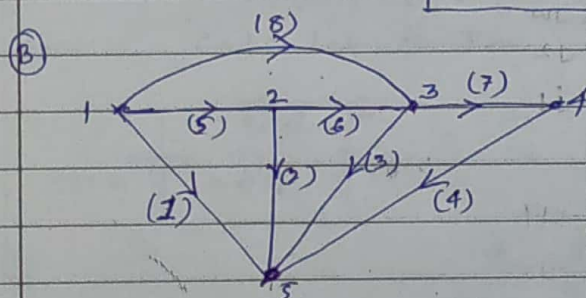
Applying KVL to Mesh 1

$$9 - 3(I_1 - I_2) - 6I_1 = 0$$

$$9I_1 - 3I_2 = 9 \quad \text{--- (2)}$$

From Mesh 2:  $I_2 = 6I_x = 6I_1$ ,  $-6I_1 - I_2 = 0$  - - (3)Solving (2) & (3):  $I_1 = -1A$ ,  $I_2 = -6A$ Writing  $V_{th}$  eqn:  $9 - 3(I_1 - I_2) + 2I_2 - V_{th} = 0$ 

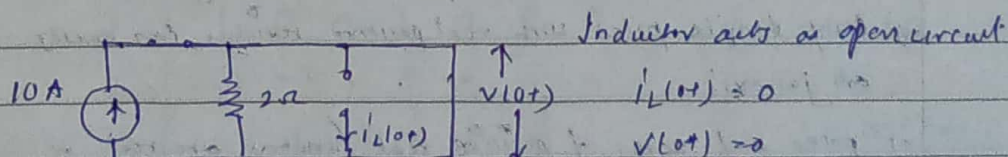
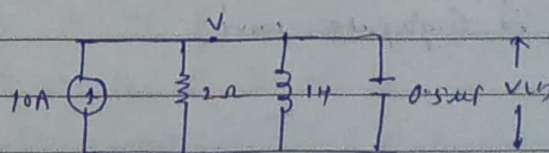
$$V_{th} = -18V$$



(C)

At  $t=0$  - no current flows through the inductor  
 and there is no voltage across capacitor.

$$i_L(0^-) = 0, \quad v_C(0^-) = 0$$

At  $t=0^+$ For  $t > 0$ Writing KCL eqn for  $t > 0$ 

$$\frac{V}{2} + \frac{1}{1} \int V dt + 0.5 \times 10^{-6} \frac{dV}{dt} - 10 = 0$$

$$\text{At } t=0^+ \quad \frac{V(0^+)}{2} + 0 + 0.5 \times 10^{-6} \frac{dV}{dt}(0^+) = 10$$

$$\frac{dV(0^+)}{dt} = 20 \times 10^6 \text{ V/s}$$

d) ABCD parameters are:  $V_1 = AV_2 - BI_2$  — (1)

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

Rewriting eqn (2)

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2 \quad \text{--- (3)}$$

$$4 \quad V_1 = AV_2 - BI_2$$

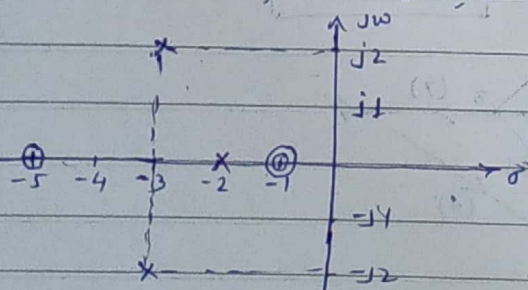
$$= AV_2 - B \left[ -\frac{1}{D} I_1 + \frac{C}{D} V_2 \right]$$

$$V_1 = \frac{B}{D} I_1 + \left[ \frac{AD - BC}{D} \right] V_2 \quad \text{--- (4)}$$

Comparing eqn (3) & (4) with std. h parameters

$$h_{11} = \frac{B}{D}, \quad h_{12} = \frac{AD - BC}{D}, \quad h_{21} = -\frac{1}{D}, \quad h_{22} = \frac{C}{D}$$

e) Function  $F(s)$  has zero at  $s = -1, -1$  &  $s = -5$  and poles at  $s = -2, s = -3 + j2, s = -3 - j2$



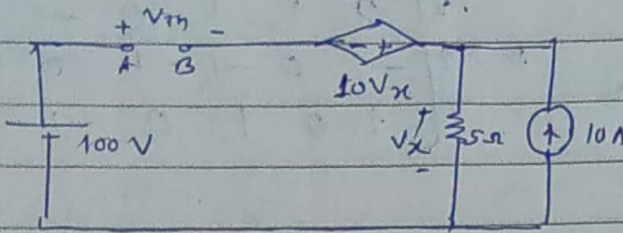
f) properties of positive real function

- i) If  $F(s)$  is positive real then  $\frac{1}{F(s)}$  is also positive real
- ii) Sum of two positive real functions is positive real
- iii) poles & zeros of positive real function cannot have positive real parts i.e. they cannot be in right half of s-plane
- iv) Only simple poles with real positive residue can exist on jω axis
- v) poles and zeros of positive real functions are real or occur in conjugate pairs



Q2A

Step 1: Calculation of  $V_{Th}$



from fig:  $V_x = 10 \times 5 = 50V$

Writing  $V_{Th}$  eqn

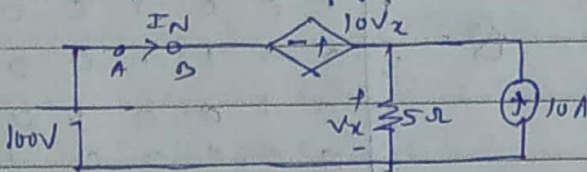
$$100 - V_{Th} + 10V_x - V_x = 0$$

$$100 - V_{Th} + 9V_x = 0$$

$$100 - V_{Th} + 9(50) = 0$$

$$V_{Th} = 550 \text{ Volt}$$

Step 2: Calculation of  $I_N$



from fig:  $V_x = 5(I_N + 10)$

Apply KVL to Mesh (1)

$$100 + 10V_x - V_x = 0$$

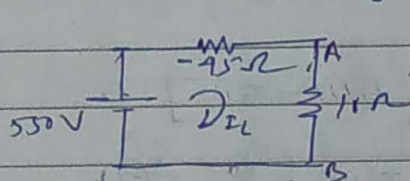
$$V_x = -100/9$$

$$-100/9 = 5I_N + 50$$

$$I_N = -550/45 \text{ A}$$

Step III: Calculation of  $R_{Th}$   $R_{Th} = \frac{V_{Th}}{I_N} = \frac{550}{-550/45} = -45 \Omega$

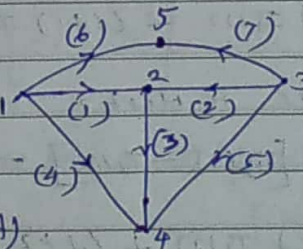
Step IV: Calculation of  $I_N$



$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{550}{-45 + 10} = \underline{\underline{-110/7 \text{ A}}}$$

B

Oriented graph



Incidence matrix (A)

Nodes	1	2	3	4	5	6	7
1	1	0	0	1	0	1	0
2	-1	-1	1	0	0	0	0
3	0	1	0	0	1	0	1
4	0	0	-1	-1	-1	0	0
5	0	0	0	0	0	-1	-1

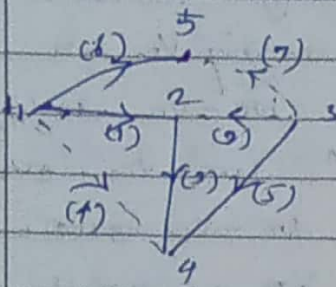
$$A_a = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

Reduced incidence matrix (A)

while eliminating last row from matrix (Aa)

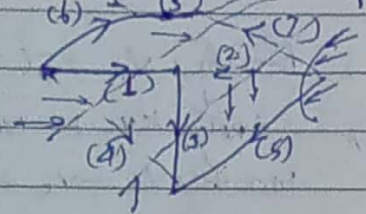
$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 \end{bmatrix}$$

Incident matrix (B)



$$B = \begin{bmatrix} 0 & 1 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & -1 & 1 \end{bmatrix}$$

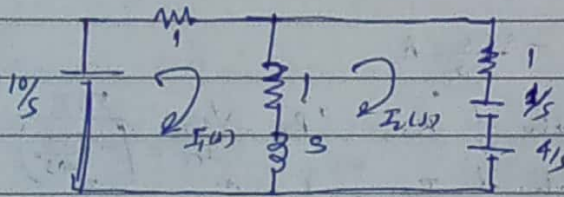
Cut-set matrix



$$Q = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

e)

for  $t > 0$  transformed network



Apply KVL to Mesh 1

$$\frac{10}{s} - I_1(s) - (1+s) [I_1(s) - I_2(s)] = 0$$

$$(s+2) I_1(s) - (s+1) I_2(s) = \frac{10}{s} \quad (1)$$

Apply KVL to Mesh 2

$$-(s+1) [I_2(s) - I_1(s)] - I_2(s) - \frac{1}{s} I_2(s) = \frac{4}{s}$$

$$-(s+1) I_1(s) + (s+2 + \frac{1}{s}) I_2(s) = \frac{4}{s} \quad (2)$$

By Cramer's rule

$$I_1(s) = \frac{A_1}{\Delta} = \frac{\begin{vmatrix} \frac{10}{s} & -(s+1) \\ -\frac{4}{s} & s+2+\frac{1}{s} \end{vmatrix}}{\begin{vmatrix} s+2 & -(s+1) \\ -(s+1) & s+2+\frac{1}{s} \end{vmatrix}} = \frac{3s+5}{s(s+1)}$$

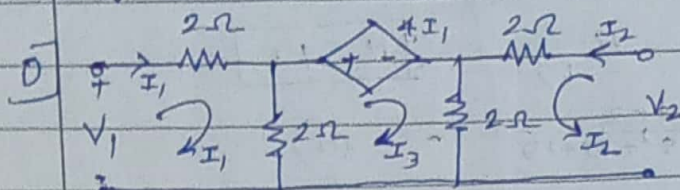
By partial fraction  $I_1(s) = \frac{A}{s} + \frac{B}{s+1}$

$$I_1(s) = \frac{5}{s} - \frac{2}{s+1} \quad \text{by inverse LT, } i_1(t) = 5 - 2e^{-t}$$

$$\text{Similarly, } I_2(s) = \frac{A_2}{\Delta} = \frac{\begin{vmatrix} s+2 & \frac{10}{s} \\ -(s+1) & -\frac{4}{s} \end{vmatrix}}{\begin{vmatrix} s+2 & -(s+1) \\ -(s+1) & s+2+\frac{1}{s} \end{vmatrix}} = \frac{3}{s+1} - \frac{2}{s+1} = \frac{1}{s+1}$$



By Inverse LT  $i_{out}(t) = 3e^{-t} - 2te^{-t}$



Applying KVL to Mesh ①

$$V_1 = 2I_1 + 2(I_1 - I_3)$$

$$V_1 = 4I_1 - 2I_3 \quad \text{--- (i)}$$

Applying KVL to Mesh 2

$$V_2 = 2I_2 + 2(I_2 + I_3)$$

$$V_2 = 4I_2 + 2I_3 \quad \text{--- (ii)}$$

Applying KVL to Mesh 3

$$-2(I_3 - I_1) - 4I_1 - 2(I_3 + I_2) = 0$$

$$I_1 + I_2 = -2I_3 \quad \text{--- (iii)}$$

put eq (iii) in (i) & (ii)

$$V_1 = 4I_1 + I_1 + I_2 = 5I_1 + I_2 \quad \text{--- (iv)}$$

$$V_2 = 4I_2 - I_1 - I_2 = -I_1 + 3I_2 \quad \text{--- (v)}$$

Compare (iv) & (v) with std Z parameters

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

To find h-parameters

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (5)(3) - (1)(-1) = 16$$

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{16}{3} \Omega, \quad h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{1}{3}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = -\frac{1}{3}, \quad h_{22} = \frac{1}{Z_{22}} = \frac{1}{3} \Omega^{-1}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{16}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

E]

$$Z(s) = \frac{(R+Ls) \frac{1}{Cs}}{R+Ls + \frac{1}{Cs}} = \frac{\frac{1}{C}(1+Rs)}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \quad \text{--- (i)}$$

from pole zero diagram

Zero is at  $s = -2$  & poles are at  $s = -1 \pm j4$

$$Z(s) = H \frac{s+2}{(s+1+j4)(s+1-j4)} = H \frac{s+2}{(s+1)^2 - (4j)^2} = H \frac{s+2}{s^2 + 2s + 17}$$

At d.c.  $2 = H \cdot \frac{2}{17} \Rightarrow H = 17$

$$Z(s) = 17 \cdot \frac{s+2}{s^2 + 2s + 17} \quad \text{--- (i)}$$

Comparing eq<sup>n</sup> (ii) with (i)  $\frac{1}{C} = 17, \frac{R}{L} = 2, \frac{1}{LC} = 17$

$$C = \frac{1}{17} F, L = 1H, R = 2\Omega$$

F) Cauer I form:

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)} = \frac{2s^2 + 8s + 6}{s^2 + 8s + 12}$$

By Continued fraction expansion

$$\begin{array}{r} s^2 + 8s + 12 \overline{) 2s^2 + 8s + 6} \quad (2 \leftarrow Z) \\ \underline{2s^2 + 16s + 24} \\ -8s + 18 \end{array}$$

$\therefore$  negative result, Continued fraction expansion of  $Y(s)$  carried out

$$Y(s) = \frac{s^2 + 8s + 12}{2s^2 + 8s + 12}$$

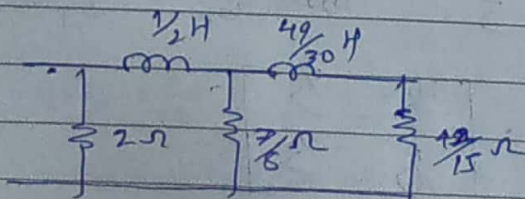
By Continued Fraction expansion

$$\begin{array}{r} 2s^2 + 8s + 12 \overline{) s^2 + 8s + 12} \quad \left( \frac{1}{2} \leftarrow Y \right) \\ \underline{s^2 + 4s + 2} \\ 4s + 10 \end{array}$$

$$\begin{array}{r} 4s + 10 \overline{) 2s^2 + 8s + 6} \quad \left( \frac{1}{2}s + 2 \right) \\ \underline{2s^2 + \frac{9}{2}s} \\ \frac{1}{2}s + 6 \end{array}$$

$$\begin{array}{r} \frac{1}{2}s + 6 \overline{) 4s + 10} \quad \left( \frac{8}{7} \leftarrow Y \right) \\ \underline{\frac{4}{7}s + \frac{48}{7}} \\ \frac{1}{7}s + \frac{14}{7} \end{array}$$

$$\begin{array}{r} \frac{1}{7}s + \frac{14}{7} \overline{) \frac{1}{2}s + 6} \quad \left( \frac{14}{11} \leftarrow Y \right) \\ \underline{\frac{1}{11}s + \frac{14}{11}} \\ \frac{1}{11}s + \frac{14}{11} \end{array}$$



Cauer I form

Cauer II form

Bringing Numerator & Denominator polynomials in ascending order of  $s$ .



Q2) Equation for

$$Z(s) = 6 + 8s + 2s^2$$

$$12 + 8s + s^2$$

By Continued Fraction expansion

$$\frac{12 + 8s + s^2}{s + 4s + \frac{1}{2}s^2} \left( \frac{1}{2} \leftarrow 2 \right)$$

$$\frac{4s + \frac{3}{2}s^2}{12 + 8s + s^2} \left( \frac{3}{5} \leftarrow 1 \right)$$

$$12 + \frac{9}{2}s$$

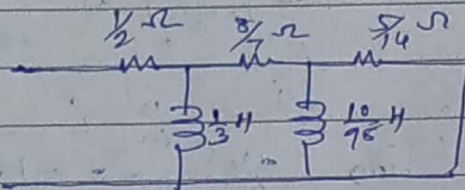
$$\frac{\frac{7}{2}s + s^2}{4s + \frac{3}{2}s^2} \left( \frac{8}{7} \leftarrow 2 \right)$$

$$\frac{1 + \frac{3}{4}s^2}{\frac{5}{14}s^2}$$

$$\frac{\frac{5}{14}s^2}{\frac{7}{2}s} + \frac{s^2}{\frac{5}{14}s^2} \left( \frac{98}{10} \leftarrow 1 \right)$$

$$\frac{\frac{7}{2}s}{\frac{5}{14}s^2} + \frac{s^2}{\frac{5}{14}s^2} \left( \frac{5}{14} \leftarrow 2 \right)$$

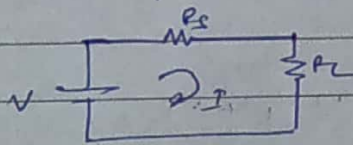
$$\frac{\frac{5}{14}s^2}{10}$$



Q3) 1

Maximum power Transfer theorem

• Max power is delivered from source to load when  $R_L = R_s$



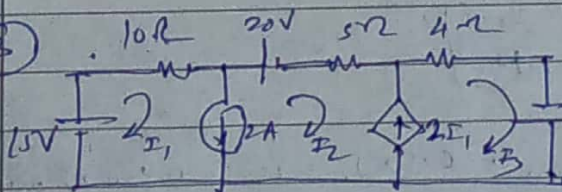
$$I = \frac{V}{R_s + R_L}$$

$$P_L = I^2 R_L = \frac{V^2 R_L}{(R_s + R_L)^2}$$

To determine value of  $R_L$  max power transferred to load

$$\frac{dP_L}{dR_L} = 0 \Rightarrow \frac{d}{dR_L} \left[ \frac{V^2 R_L}{(R_s + R_L)^2} \right] = 0 \Rightarrow \boxed{R_L = R_s}$$

Q4) 1



Mesh 1, 2, 3 will form

Supermesh

Writing current eq<sup>n</sup> for supermesh

$$I_1 - I_2 = 2 \quad (i)$$

$$I_3 - I_2 = 2I_1 \quad (ii)$$

$$\therefore 2I_1 + I_2 - I_3 = 0$$

Applying KVL to outer loop of supermesh

$$15 - 10I_1 - 20 - 5I_2 - 4I_3 + 40 = 0$$

$$10I_1 + 5I_2 + 4I_3 = 35 \quad (iii)$$

Solving eq<sup>n</sup> (i), (ii) & (iii)

$$I_1 = 1.96A, \quad I_2 = -0.04A, \quad I_3 = 3.87A$$

$$\boxed{P_{10\Omega} = I_1^2 R = 1.96A^2 \times 10\Omega}$$