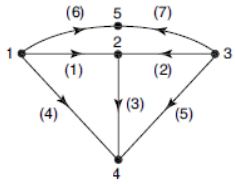
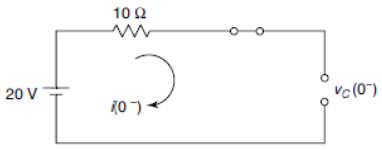
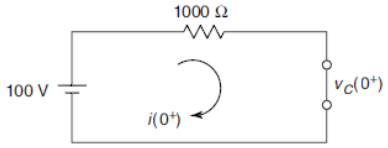
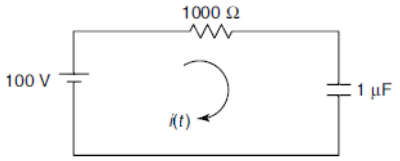
(Affiliated to University of Mumbai)	***End Semester Examination (R-24/R-25) SH 2026 Answer Key with marking scheme***
Branch: EXTC	Course: Network Theory
Year/ Semester: SE/III	Course code: ETC304
Time: 03 hours	Marks: 80

		Marks
Q. 1	All questions are compulsory.	
A.	Definition of Tree with diagram – 1 mark Definition of Twig with diagram – 1 mark Definition of Link with diagram – 1 mark Definition of Path with diagram – 1 mark Definition of Loop with diagram – 1 mark	5
B.	**Solution** Applying KVL to Mesh 1, $-30-6I_1-15(I_1-I_2)=0$ $21I_1-15I_2=-30$ …(i) Applying KVL to Mesh 2, $-10(I_2-I_3)-15(I_2-I_1)+50-5I_2=0$ $-15I_1+30I_2-10I_3=50$ …(ii) For Mesh 3, $I_3=1$ …(iii) Solving Eqs (i), (ii) and (iii), $I_1=0$ $I_2=2$ A $I_3=1$ A KVL equation for Mesh 1 – 1 mark KVL equation for Mesh 2 – 1 mark Current source constraint for Mesh 3 – 1 mark Solving simultaneous equations – 1 mark Final mesh currents – 1 mark	5
C.	**Solution** For drawing the oriented graph, 1. replace all resistors, inductors and capacitors by line segments, 2. replace all voltage sources by short circuits and current source by an open circuit, 3. assume directions of branch currents arbitrarily, and 4. number all the nodes and branches. The oriented graph is shown in Fig.  (a) Incidence Matrix (A) Nodes ↓ | Branches → 1 2 3 4 5 6 7 1 1 0 0 1 0 1 0 2 −1 −1 1 0 0 0 0 3 0 1 0 0 1 0 1 4 0 0 −1 −1 −1 0 0 5 0 0 0 0 0 −1 −1 $A_a=\begin{bmatrix}1&0&0&1&0&1&0\\-1&-1&1&0&0&0&0\\0&1&0&0&1&0&1\\0&0&-1&-1&-1&0&0\\0&0&0&0&0&-1&-1\end{bmatrix}$ Eliminating the last row from the matrix A_a, we get the incidence matrix A. $A=\begin{bmatrix}1&0&0&1&0&1&0\\-1&-1&1&0&0&0&0\\0&1&0&0&1&0&1\\0&0&-1&-1&-1&0&0\end{bmatrix}$ Identification of nodes and branches – 1 mark Correct oriented graph – 2 marks Complete incidence matrix – 1 mark Reduced incidence matrix – 1 mark	5

D.	(a) In the given polynomial, the term s^2 is missing and it is neither an even nor an odd polynomial. Hence, it is not Hurwitz. (b) Polynomial $s^6+5s^5+4s^4-3s^3+2s^2+s+3$ is not Hurwitz as it has a term $(-3s^3)$ which has a negative coefficient. Identification of missing coefficient and justification for polynomial (a) – 2 marks Identification of negative coefficient for polynomial (b) – 1 mark Justification of Hurwitz condition for polynomial (b) – 2 marks	5
E.	**Solution** By voltage division rule, $V_2(s)=V_1(s)\dfrac{\frac{1}{Cs}}{R+\frac{1}{Cs}}=V_1(s)\dfrac{1}{RCs+1}=V_1(s)\dfrac{\frac{1}{RC}}{s+\frac{1}{RC}}$ Voltage transfer function $\dfrac{V_2(s)}{V_1(s)}=\dfrac{\frac{1}{RC}}{s+\frac{1}{RC}}$ Impedance expressions for R and C – 1 mark Application of voltage division rule – 2 marks Simplification of transfer function – 1 mark Final voltage transfer function – 1 mark	5
F.	**Solution** At $t=0^-$, the network is shown  At $t=0^-$, the network attains steady-state condition. Hence, the inductor acts as a short circuit and the capacitor acts as an open circuit. $v_C(0^-)=20\text{ V}$ $i(0^-)=0$ Since the current through the inductor and the voltage across the capacitor cannot change instaneously, $v_C(0^+)=20\text{ V}$ $i(0^+)=0$ Equivalent circuit before switching – 1 mark Initial inductor current and capacitor voltage – 1 mark Application of continuity conditions – 1 mark Formulation of circuit equation for t>0 – 1 mark Final current expression i(t) – 1 mark	5
Q.2A	**Solution** At $t=0^-$, the capacitor is uncharged. $v_C(0^-)=0$ $i(0^-)=0$ At $t=0^+$, the network is shown in Fig. 10.7. At $t=0^+$, the capacitor acts as a short circuit. $v_C(0^+)=0$ $i(0^+)=\dfrac{100}{1000}=0.1\text{ A}$ 	10

For $t > 0$, the network is shown in Fig.
Writing the KVL equation for $t > 0$,



$$100 - 1000i - \frac{1}{1\times10^{-6}}\int_0^t i\,dt = 0 \qquad \ldots(i)$$

Differentiating Eq. (i),

$$0 - 1000\frac{di}{dt} - 10^6 i = 0$$

$$\frac{di}{dt} = -\frac{10^6}{1000}i \qquad \ldots(ii)$$

At $t = 0^+$, $\quad \frac{di}{dt}(0^+) = -\frac{10^6}{1000}i(0^+) = -\frac{10^6}{1000}(0.1) = -100 \text{ A/s}$

Differentiating Eq. (ii),

$$\frac{d^2i}{dt^2} = -\frac{10^6}{1000}\frac{di}{dt}$$

At $t = 0^+$, $\quad \frac{d^2i}{dt^2}(0^+) = -\frac{10^6}{1000}\frac{di}{dt}(0^+) = -\frac{10^6}{1000}(-100) = 10^5 \text{ A/s}^2$

Initial capacitor voltage and current – 2 marks
KVL equation for t>0 – 2 marks
First differentiation of circuit equation – 2 marks
Calculation of di/dt at $t=0^+$ – 2 marks
Calculation of d^2i/dt^2 at $t=0^+$ – 2 marks

OR

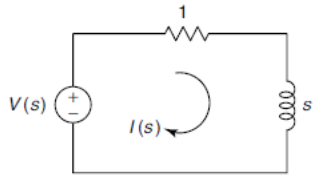
Q.2B. 10

Solution

$$v(t) = u(t) - u(t-1)$$

$$V(s) = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1-e^{-s}}{s}$$

For $t > 0$, the transformed network is shown in Fig.
Applying KVL to the mesh,



$$V(s) - I(s) - sI(s) = 0$$

$$I(s) = \frac{V(s)}{s+1}$$

$$= \frac{1-e^{-s}}{s(s+1)}$$

$$= \frac{1}{s(s+1)} - \frac{e^{-s}}{s(s+1)}$$

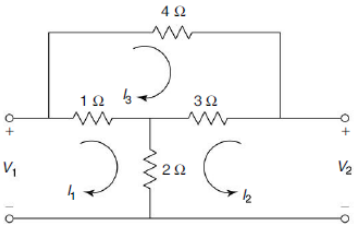
$$= \frac{1}{s} - \frac{1}{s+1} - \frac{e^{-s}}{s} + \frac{e^{-s}}{s+1}$$

Taking inverse Laplace transform,

$$i(t) = u(t) - e^{-t}u(t) - u(t-1) + e^{-(t-1)}u(t-1)$$

$$= (1-e^{-t})u(t) - [1-e^{-(t-1)}]u(t-1) \qquad \text{for } t > 0$$

Laplace transform of input voltage – 2 marks
Frequency-domain equivalent circuit and KVL equation – 2 marks
Expression for I(s) – 2 marks
Partial fraction expansion – 2 marks
Inverse Laplace transform and final i(t) – 2 marks

Q.3A	**Solution** The network is redrawn as shown in Fig. Applying KVL to Mesh 1, $V_1 - 1(I_1 - I_3) - 2(I_1 + I_2) = 0$ $V_1 = 3I_1 + 2I_2 - I_3$ …(i) Applying KVL to Mesh 2, $V_2 - 3(I_2 + I_3) - 2(I_1 + I_2) = 0$ $V_2 = 2I_1 + 5I_2 + 3I_3$ …(ii) Applying KVL to Mesh 3, $-4I_3 - 3(I_2 + I_3) - 1(I_3 - I_1) = 0$ $I_1 - 3I_2 + 8I_3 = 0$ $I_3 = \frac{1}{8}I_1 - \frac{3}{8}I_2$ …(iii)  Substituting Eq. (iii) in Eq. (i), $V_1 = 3I_1 + 2I_2 - \left(\frac{1}{8}I_1 - \frac{3}{8}I_2\right)$ $= \frac{23}{8}I_1 + \frac{19}{8}I_2$ …(iv) Substituting Eq. (iii) in Eq. (ii), $V_2 = 2I_1 + 5I_2 + 3\left(\frac{1}{8}I_1 - \frac{3}{8}I_2\right)$ $= \frac{19}{8}I_1 + \frac{31}{8}I_2$ …(v) Comparing Eqs (iv) and (v) with Z-parameter equations, $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{23}{8} & \frac{19}{8} \\ \frac{19}{8} & \frac{31}{8} \end{bmatrix}$ Since $Z_{11} \neq Z_{22}$, the network is not symmetrical. Since $Z_{12} = Z_{21}$, the network is reciprocal. KVL equations for Mesh 1 and Mesh 2 – 2 marks KVL equation for Mesh 3 – 2 marks Substitution and simplification of equations – 2 marks Determination of four Z-parameters and matrix representation – 2 marks Verification of symmetry and reciprocity conditions – 2 marks	10
	OR	
Q.3B	Z-parameter equations – 2 marks Derivation of Z11 and Z21 – 2 marks Derivation of Z12 and Z22 – 2 marks Z-parameter equivalent circuit diagram – 2 marks Verification and Justification of symmetry and reciprocity condition – 2 marks	10
Q.4A	**Solution** From Fig. $I_1 = \frac{V_1 - 50}{20 + 10} = \frac{V_1 - 50}{30}$ …(i) Assume that the currents are moving away from the node. Applying KCL at Node 1, $2 = \frac{V_1}{5} + \frac{V_1 + 30\,I_1}{10} + \frac{V_1 - 50}{30}$ $2 = \frac{V_1}{5} + \frac{V_1 + 30\left(\frac{V_1 - 50}{30}\right)}{10} + \frac{V_1 - 50}{30}$ $2 = \frac{V_1}{5} + \frac{2V_1 - 50}{10} + \frac{V_1 - 50}{30}$ …(ii) Solving Eq. (ii), $V_1 = 20$ V Voltage across the 5 Ω resistor = 20 V Identification of node voltage and branch currents – 2 marks Expression for controlling current I_1 – 2 marks Application of KCL at the node – 3 marks Simplification and solution of equation – 2 marks Final voltage across 5 Ω resistor – 1 mark	10

OR

Q.4B

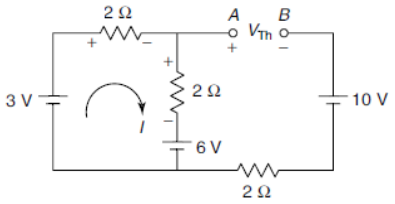
Solution 10

Step I Calculation of V_{Th}

Applying KVL to the mesh,

$$3 - 2I - 2I - 6 = 0$$

$$I = -0.75\,\text{A}$$



Writing the V_{Th} equation,

$$6 + 2I - V_{Th} - 10 = 0$$

$$V_{Th} = 6 + 2I - 10 = 6 + 2(-0.75) - 10 = -5.5\,\text{V}$$

$$= 5.5\ \text{V (terminal } \boldsymbol{B} \text{ is positive w.r.t } A)$$

Step II Calculation of R_{Th} (Fig. 3.367)

$$R_{Th} = (2 \parallel 2) + 2 = 3\,\Omega$$

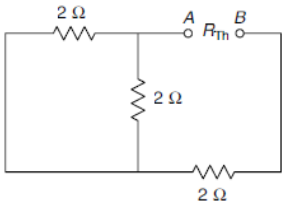


Fig. 3.367

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 3\,\Omega$$

Step IV Calculation of P_{max} (Fig. 3.368)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(5.5)^2}{4 \times 3} = 2.52\,\text{W}$$

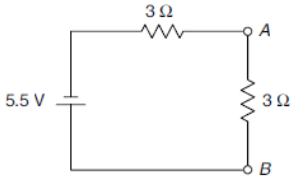


Fig. 3.368

Calculation of Thevenin voltage V_{Th} – 3 marks
Calculation of Thevenin resistance R_{Th} – 3 marks
Determination of load resistance R_L – 2 marks
Calculation of maximum power P_{max} – 2 marks

Q.5A 10

(a) $$F(s) = \frac{N(s)}{D(s)} = \frac{s^2 + 1}{s^3 + 4s} = \frac{(s + j1)(s - j1)}{s(s + j2)(s - j2)}$$

The function $F(s)$ has poles at $s = 0$, $s = -j2$ and $s = j2$ and zeros at $s = -j1$ and $s = j1$ as shown in Fig. 16.4.

Thus, all the poles and zeros are on the $j\omega$ axis.

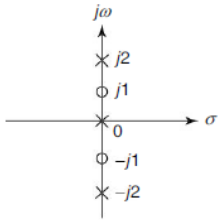


Fig. 16.4

(b) The poles on the $j\omega$ axis are simple. Hence, residue test is carried out.

$$F(s) = \frac{s^2 + 1}{s^3 + 4s} = \frac{s^2 + 1}{s(s^2 + 4)}$$

By partial-fraction expansion,

$$F(s) = \frac{K_1}{s} + \frac{K_2}{s + j2} + \frac{K_2^*}{s - j2}$$

The constants K_1, K_2 and K_2* are called residues.

$$K_1 = s\,F(s)|_{s=0} = \left.\frac{s^2 + 1}{s^2 + 4}\right|_{s=0} = \frac{1}{4}$$

$$K_2 = (s + j2)F(s)|_{s=-j2} = \left.\frac{s^2 + 1}{s(s - j2)}\right|_{s=-j2} = \frac{-4 + 1}{(-j2)(-j2 - j2)} = \frac{3}{8}$$

$$K_2^* = K_2 = \frac{3}{8}$$

Thus, residues are real and positive.

(c) Even part of $N(s) = m_1 = s^2 + 1$

Odd part of $N(s) = n_1 = 0$

Even part of $D(s) = m_2 = 0$

Odd part of $D(s) = n_2 = s^3 + 4s$

$$A(\omega^2) = m_1 m_2 - n_1 n_2 \big|_{s=j\omega} = (s^2+1)(0) - (0)(s^3+4s)\big|_{s=j\omega} = 0$$

$A(\omega^2)$ is zero for all $\omega \geq 0$.

Since all the three conditions are satisfied, the function is positive real.

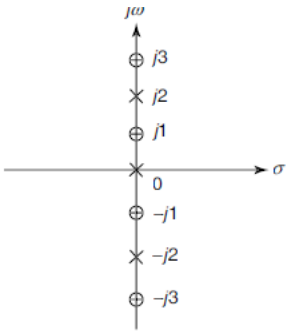
Identification of poles and zeros – 2 marks
Verification of poles in the left half-plane or on the imaginary axis – 2 marks
Verification of simple imaginary-axis poles and positive residues – 3 marks
Verification of real-part condition – 2 marks
Final conclusion – 1 mark

OR

Q.5B

10

Solution The function $Z(s)$ has poles at $s = 0$ and $s = \pm j2$ and zeros at $s = \pm j1$ and $s = \pm j3$ as shown in Fig. 16.23.



From the pole-zero diagram, it is clear that poles and zeros are simple and lie on the $j\omega$ axis. Poles and zeros are interlaced. Hence, the given function is an LC function.

Foster I Form The Foster I form is obtained by partial-fraction expansion of the impedance function $Z(s)$. But degree of numerator is greater than degree of denominator. Hence, division is first carried out.

$$Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)} = \frac{4s^4+40s^2+36}{s^3+4s}$$

$$s^3+4s)\ 4s^4+40s^2+36\ (4s$$

$$\frac{4s^4+16s^2}{24s^2+36}$$

$$Z(s) = 4s + \frac{24s^2+36}{s^3+4s} = 4s + \frac{24s^2+36}{s(s^2+4)}$$

By partial-fraction expansion,

$$Z(s) = 4s + \frac{K_0}{s} + \frac{K_1}{s+j2} + \frac{K_1^*}{s-j2} = 4s + \frac{K_0}{s} + \frac{2K_1 s}{s^2+4}$$

where

$$K_0 = sZ(s)\big|_{s=0} = \frac{4(1)(9)}{4} = 9$$

$$K_1 = \frac{(s^2+4)Z(s)}{2s}\bigg|_{s^2=-4} = \frac{4(-4+1)(-4+9)}{2(-4)} = \frac{15}{2}$$

$$Z(s) = 4s + \frac{9}{s} + \frac{15s}{s^2+4}$$

The first term represents the impedance of an inductor of 4 H. The second term represents the impedance of a capacitor of $\frac{1}{9}$ F. The third term represents the impedance of a parallel LC network.

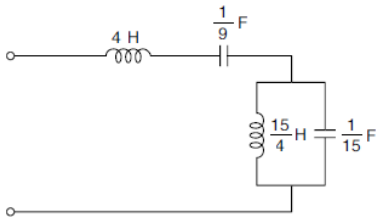
For a parallel LC network,

$$Z_{LC}(s) = \frac{\left(\frac{1}{C}\right)s}{s^2 + \frac{1}{LC}}$$

By direct comparison,

$$C = \frac{1}{15}\text{F}$$

$L = \frac{15}{4}$ H

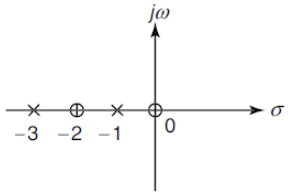


Identification of poles and zeros and verification of interlacing – 2 marks
Polynomial division and partial fraction expansion – 3 marks
Calculation of circuit element values – 3 marks
Foster Form I equivalent circuit diagram – 2 marks

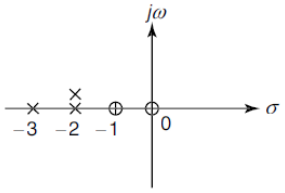
Q.6A

10

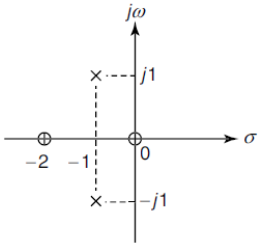
(a) The function $F(s)$ has zeros at $s = 0$ and $s = -2$ and poles at $s = -1$ and $s = -3$. The pole-zero plot is shown in Fig. 12.42.



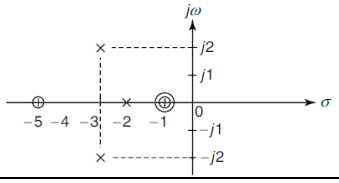
(b) The function $F(s)$ has zeros at $s = 0$ and $s = -1$ and poles at $s = -2, -2$ and $s = -3$. The pole-zero plot is shown in Fig. 12.43.



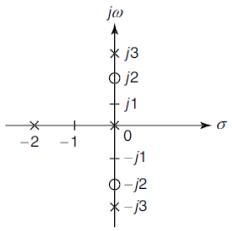
(c) The function $F(s)$ has zeros at $s = 0$ and $s = -2$ and poles at $s = -1 - j1$ and $s = -1 + j1$. The pole-zero plot is shown in Fig. 12.44.



(d) The function $F(s)$ has zeros at $s = -1, -1$ and $s = -5$ and poles at $s = -2$, $s = -3 + j2$ and $s = -3 - j2$. The pole-zero plot is shown in Fig. 12.45.



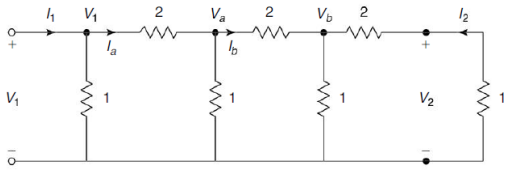
(e) The function $F(s)$ has zeros at $s = j2$ and $s = -j2$ and poles at $s = -2$, $s = j3$ and $s = -j3$. The pole-zero plot is shown in Fig. 12.46.



For each of the five functions (a), (b), (c), (d) and (e): Correct identification of poles and zeros – 1 mark, Correct pole-zero plot – 1 mark

OR

Q.6B



$$I_2 = -\frac{V_2}{1} = -V_2$$

$$V_b = -3I_2 = 3V_2$$

$$I_b = \frac{V_b}{1} + \frac{V_b}{3} = \frac{4}{3}V_b = 4V_2$$

$$V_a = 2I_b + V_b = 8V_2 + 3V_2 = 11V_2$$

$$I_a = \frac{V_a}{1} + I_b = 11V_2 + 4V_2 = 15V_2$$

$$V_1 = 2I_a + V_a = 30V_2 + 11V_2 = 41V_2$$

$$I_1 = \frac{V_1}{1} + I_a = 41V_2 + 15V_2 = 56V_2$$

$$\frac{V_2}{V_1} = \frac{1}{41}$$

$$\frac{V_2}{I_1} = \frac{1}{56}\,\Omega$$

$$\frac{I_2}{V_1} = -\frac{1}{41}\ \text{℧}$$

$$\frac{I_2}{I_1} = -\frac{1}{56}$$

Application of KCL and calculation of intermediate node voltages – 3 marks
Derivation of input voltage and current relationships – 3 marks
Calculation of voltage ratio V_2/V_1 – 1 mark
Calculation of transfer impedance V_2/I_1 – 1 mark
Calculation of transfer admittance I_2/V_1 – 1 mark
Calculation of current ratio I_2/I_1 – 1 mark

10

*********** All the Best***********