Signals & Systems. Solution Set 3 Sem III ExTC

ETC305

Q.1. a). $x(t) = e^{-t}u(t)$

Non-Periodic Signal, Energy Signal 01

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_0^{\infty} e^{-2t} \, dt \quad \text{as } u(t)=1.$$ 01

$$E = \int_0^{\infty} e^{-2t} dt = \left[\frac{e^{-2t}}{-2}\right]_0^{\infty} = -\frac{1}{2}[0-1] = \frac{1}{2}$$ 02

$P = 0$. 01

b) $y(t) = x(2t)$

Linear 01
Non Causal, 01
Dynamic 01
Stable 01
time variant 01

c) ROC :- Region of convergence

z-xform → values of z for which $X(z)$ exists 02
Laplace → values of s for which $X(s)$ exists 02

Eg :- $X(z) = \frac{1}{1-az^{-1}}$ for $a^n u(n)$ ROC: $|z| > a$ 01

Eg. $X(s) = \frac{1}{s+a}$ for $e^{-at}u(t)$ ROC. $Re(s) > -a$

d) $X(s) = \int_0^2 1\, e^{-st} dt$ 02

$$= \left[\frac{e^{-st}}{-s}\right]_0^2 = -\frac{1}{s}\left[e^{-2s}-1\right] = \frac{1-e^{-2s}}{s}$$ 02

ROC :- $Re(s) > 0$ 01

e) Two properties :-

Property 1 — 01 Equations
property 2 01 Equation } → 03

f) BIBO → Bounded I/p bounded o/p, Explanation $02\frac{1}{2}$

ROC :- Region of Convergence Explanation $02\frac{1}{2}$

Q.2 a) $h(t) = e^{-2t} u(t)$ $x(t) = u(t)$.

(2)

$$y(t) = \int h(\tau)\, x(t-\tau)\, d\tau$$ 02

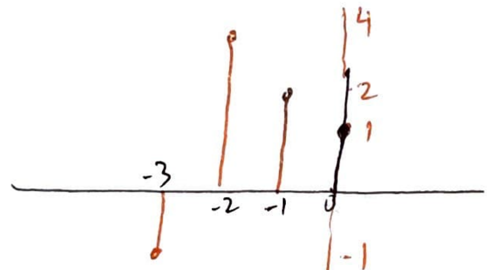
$$= \int e^{-2\tau} u(\tau)\, u(t-\tau)\, d\tau$$ $u(\tau) = 1 \quad \tau \geq 0$ 02

$$= \int_0^t e^{-2\tau} \cdot 1\, d\tau$$ $u(t-\tau) = 1 \quad t-\tau \geq 0, \; t \geq \tau$ 02

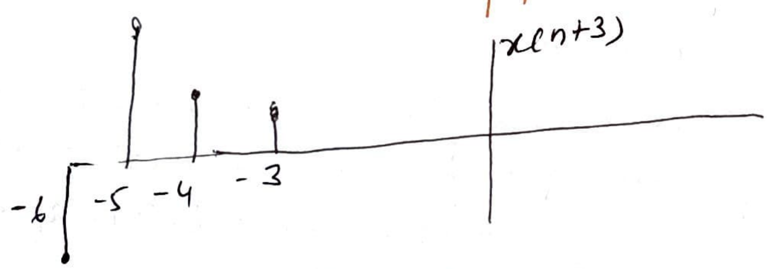
$$y(t) = \left[\frac{e^{-2\tau}}{-2}\right]_0^t = -\frac{1}{2}\left[e^{-2t} - 1\right] = \frac{1}{2}\left[1 - e^{-2t}\right]$$ 02

$$y(t) = \frac{1}{2}\left[1 - e^{-2t}\right]$$ 02

b) $x(n)$



01 for each



& so on.

02

c) $$X(s) = \frac{6(s)}{(s^2+10s+34)} = \frac{6(s)}{(s^2+2.5s+25+9)}$$

$$X(s) = \frac{6(s)}{(s^2+2.5s+5^2+3^2)} = \frac{6(s+5)-30}{[(s+5)^2+3^2]}$$ 02

$$X(s) = \frac{6(s+5)-30}{(s+5)^2+3^2} = \frac{6(s+5)}{(s+5)^2+3^2} - \frac{10 \cdot 3}{(s+5)^2+3^2}$$ 02

$$x(t) = 6e^{-5t}\cos 3t - 10 \cdot e^{-5t} \sin 3t$$ 02

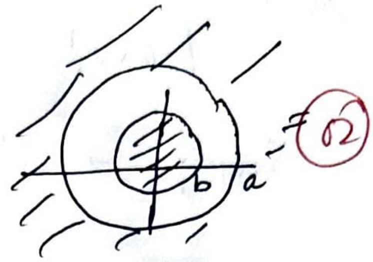
ROC $- \; \mathrm{Re}(s) > -5$ 02

3

d) $x(n) = a^n u(n) + b^n u(-n-1)$

$$X(z) = \frac{1}{1-az^{-1}} + \frac{-1}{1-bz^{-1}} \quad 02$$

$|z| > a \qquad |z| < b.$

i)  02

If ① $a > b$ ROC: – 02

② $b > a$ ROC: – $a < |z| < b$ 02

ii)  02

e) $x(t) = \cos 2\pi f_c t.$

$$x(t) = \frac{e^{-j2\pi f_c t} + e^{j2\pi f_c t}}{2} \quad 02$$

$$X(f) = \int \left(\frac{e^{-j2\pi f_c t} + e^{j2\pi f_c t}}{2} \right) e^{-j2\pi f t}\, dt \quad 02$$

$$= \int \frac{e^{-j2\pi (f+f_c)t} + e^{-j2\pi (f-f_c)t}}{2}\, dt$$

02

$$= \frac{1}{2} \left[\int e^{-j2\pi (f+f_c)t}\, dt + \int e^{-j2\pi (f-f_c)t}\, dt \right.$$

$$= \frac{1}{2} \left[\delta(f+f_c) + \delta(f-f_c) \right] \quad 04$$

f). $h(n) = h_1(n) * h_2(n) = \sum h_1(k)\, h_2(n-k)$ 03

$$= \sum (1/2)^k (1/4)^{n-k} \quad 02$$

$$= \sum_{0}^{n} \left(\frac{1}{2}\right)^k (1/4)^{n-k} = \left(\frac{1}{4}\right)^n \sum \left(\frac{1}{2} \cdot 4\right)^k \quad 02$$

$$= (1/4)^n \sum_{0}^{n} (2)^k = (1/4)^n \left[\frac{1(2^n - 1)}{2-1} \right] = \left(\frac{1}{2}\right)^n - (1/4)^n$$

03

Q3.9).

$$X(z) = \frac{1}{1-0.5z^{-1}} + \frac{-1}{1-0.8z^{-1}}$$ 02

Simplify → $$\frac{1-0.8z^{-1}-1+0.5z^{-1}}{(1-0.5z^{-1})(1-0.8z^{-1})}$$ 03

$$= \frac{1-0.3z^{-1}-1}{(1-0.5z^{-1})(1-0.8z^{-1})}$$ 02

ROC:- $|z| > 0.5$ $\quad |z| < 0.8$ 03

b) $x(0^+) = \lim_{z\to\infty} X(z) = \frac{(1+\frac{1}{\infty})}{(1-\frac{0.5}{\infty})(1+\frac{3}{\infty})} = 1$ 02

$x(\infty) = \lim_{z\to 1} (1-z^{-1})X(z) = \frac{(1+z^{-1})(1-z^{-1})}{(1-0.5z^{-1})(1+3z^{-1})} = 0$ 03

c) $X(s) = \frac{3s+7}{(s+4)(s-3)} = \frac{A}{s+4} + \frac{B}{s-3}$ 01

$X(s) = \frac{5/7}{s+4} + \frac{16/7}{s-3}$ 01

i) ROC $Re(s) > 3$. $\quad x(t) = 5/7\, e^{-4t}u(t) + 16/7\, e^{3t}u(t)$ 01

ii) $Re(s) < -4$ $\quad x(t) = -5/7\, e^{-4t}u(-t) - 16/7\, e^{+3t}u(-t)$ 01

iii) $-4 < Re(s) < 3$ $\quad x(t)\ +5/7\, e^{-4t}u(t) - 16/7\, e^{+3t}u(-t)$ 01

D) $h(t) = e^{0.5t}u(t)$

① Stability condn. $\int |h(t)|\,dt < \infty$ 03

$= \int_0^\infty |e^{0.5t}|\,dt = \infty.$ → not stable

② Causal $\quad h(t) = e^{0.5t}u(t)$ → causal 02

(5)

E.

(1) $x(t+T) = \sin 4(t+T) = \sin(4t + 2\pi)$

$4T = 2\pi$ (02)

$T = 2\pi/4$ $\qquad \underline{T = \pi/2}$

→ Periodic with $T = \pi/2$

(2) $x(n) = x(n+N) = \cos 3\pi(n+N) = \cos(3\pi n + 2\pi k)$

$3\pi N = 2\pi k$ (03)

$\frac{N}{k} = \frac{2}{3} = \frac{int}{int}$ → Periodic

for $k = 3$ $\quad \underline{N = 2}$ → Period.

F.) $h(n) = 0.4\,\delta(n+2) + 0.2\,\delta(n+1) + 0.1\,\delta(n) + 0.2\,\delta(n-1) + 0.4\,\delta(n-2).$

$H(\omega) = 0.4e^{j2\omega} + 0.2e^{j\omega} + 0.1 + 0.2e^{-j\omega} + 0.4e^{-j2\omega}.$ (01)

$H(\omega) = 0.4(e^{j2\omega} + e^{-j2\omega}) + 0.2(e^{j\omega} + e^{-j\omega}) + 0.1$ (01)

$= 0.4(2\cos 2\omega) + 0.2(2\cos\omega) + 0.1$ (01)

$= 0.4$

$H(\omega) = 0.8\cos 2\omega + 0.4\cos\omega + 0.1$ 01

$|H(\omega)| = H(\omega)$ as it is real. 01

$\angle H(\omega) = 0.$