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1A characteristic equation : $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ (1)
eigenvalues : $\lambda_1 = 1$ $\lambda_2 = 2$ $\lambda_3 = 3$ (2)
eigenvectors : $x_1 = (0,1,1)$, $x_2 = (1,1,1)$, $x_3 = (1,0,1)$ (2)

~~1E~~ 1B : theory based question

1C $P(n) : \left(7^{n+2} + 8^{2n+1}\right) \mid 57, \quad \forall n \in W$
$P(0)$ is true as $7^2 + 8 = 57 \mid 57$ — (1)
Assume $P(k)$ is true : $\left(7^{k+2} + 8^{2k+1}\right) \mid 57$ — (1)
For $P(k+1)$: $7^{(k+1)+2} + 8^{2(k+1)+1} = 7^{k+3} + 8^{2k+3}$
$= 7 \cdot 7^{k+2} + 64 \cdot 8^{2k+1} = 7 \cdot 7^{k+2} + 7 \cdot 8^{2k+1} + 57 \cdot 8^{2k+1}$ (3)
$= 7\left(7^{k+2} + 8^{2k+1}\right) + 57 \cdot 8^{2k+1}$ is divisible by 57.

1D. $n = 4k \Rightarrow n^2 = 16k^2 = 4(4k^2) \equiv 0 \bmod 4$ — (1)
$n = 4k+1 \Rightarrow n^2 = 16k^2 + 8k + 1 = 4(4k^2 + 2k) + 1 \equiv 1 \bmod 4$ (2)
$n = 4k+2 \Rightarrow n^2 = 16k^2 + 16k + 4 \equiv 0 \bmod 4$
$n = 4k+3 \Rightarrow n^2 = 16k^2 + 24k + 9 \equiv 4(4k^2 + 6k + 2) + 1 \equiv 1 \bmod 4.$
(2)

1E (i) $z^3 = (x^3 - 3xy^2) + i(3x^2y - y^3) = u + iv$
$f'(z) = 3z^2.$
(2)
(ii) $g(z) = \sin z = \sin x \cosh y + i \cos x \cdot \sinh y = u + iv$
$\cancel{f'(z)} = g'(z) = \cos z.$
(3).

1F (i) $I = \frac{2}{3}(i-1)$ (2) (ii) $I = \frac{2}{3}(i-1)$ (3).

2A characteristic equation : $(\lambda-1)(\lambda-2)(\lambda-3) = 0$
eigenvalues : $\lambda_1 = 1$ $\lambda_2 = 2$ $\lambda_3 = 3$ (6)
eigenvectors : $x_1 = (4,3,2)$ $x_2 = (3,2,1)$ $x_3 = (2,1,1)$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad M = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$ (4)

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2B $A'A = \begin{bmatrix} 25 & 7 \\ 7 & 25 \end{bmatrix}$

eigenvalues : $\lambda_1 = 32$ $\lambda_2 = 18$

$\sqrt{\lambda_1} = 4\sqrt{2}$ $\sqrt{\lambda_2} = 3\sqrt{2}$ (2)

eigenvectors : $x_1 = (1, 1)$ $x_2 = (1, -1)$

$V_1 = (1/\sqrt{2}, 1/\sqrt{2})$ $V_2 = (1/\sqrt{2}, -1/\sqrt{2})$

$u_1 = \frac{1}{4\sqrt{2}} A V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ (4)

$\|u_1\| = 1$, $\|u_2\| = 1$

$\therefore A = U D V' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$ (9)

2C (i)

p	q	r	$p \wedge q$ (A)	(A) $\vee r$ (B)	(A) $\vee \sim r$ (C)	(B) $\wedge$ (C)
T	T	T	T	T	T	T
T	T	F	T	~~F~~ T	T	T
T	F	T	F	~~F~~ T	F	F
T	F	F	F	F	T	F
F	T	T	F	T	F	F
F	T	F	F	F	T	F
F	F	T	F	T	F	F
F	F	F	F	F	T	F

given statement is contingency (05)

~~2D~~ (ii) $f^{-1}(x) = \frac{-3x-1}{2-x}$, $x \neq 2$ (05)

2 D. $\gcd(493, 680) = 17$
$\gcd(408, 17) = 17$ $\Rightarrow \gcd(408, 493, 680) = 17.$ — (6)

$17 = -6 \cdot 408 + 5 \cdot 493 + 0 \cdot 680.$ — (4)

2E $U = u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$

$U_x = \phi_1(x, y) = \frac{2 + e^{y}\sin x - e^{-y}\sin x - e^{y}\cos x - e^{-y}\cos x}{(2\cos x - e^{y} - e^{-y})^2}$ (3)

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$$u_y = \phi_2(x,y) = \frac{e^{-y}\cos x + e^{y}\cos x + e^{y}\sin x - e^{-y}\sin x - 2}{(2\cos x - e^{y} - e^{-y})^2} \quad (3)$$

$$f'(z) = \frac{1}{4}\operatorname{cosec}^2\frac{z}{2} \Rightarrow f(z) = \frac{-1}{2}\cos\frac{z}{2} + c. \quad (4)$$

2F $f(z) = \frac{1/2}{z+1} + \frac{1/2}{z-3}$ (2)

when $|z|<1$: $f(z) = \frac{1}{2}\frac{1}{1+z} \cancel{\frac{1}{\cancel{z}}\frac{1}{1-}} - \frac{1}{6}\frac{1}{1-z/3}$

$$= \frac{1}{2}\left[1 - z + z^2 + \cdots\right] - \frac{1}{6}\left[1 + \frac{z}{3} + \frac{z^2}{9} + \cdots\right] \quad (2)$$

when $1<|z|<3$:

$$f(z) = \frac{1}{2z}\frac{1}{1+1/z} - \frac{1}{6}\frac{1}{1-z/3}$$

$$= \frac{1}{2z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right] - \frac{1}{6}\left[1 + \frac{z}{3} + \frac{z^2}{9} + \cdots\right] \quad (3)$$

when $|z|>3$:

$$f(z) = \frac{1}{2z}\frac{1}{1+1/z} + \frac{1}{2z}\frac{1}{1-3/z}$$

$$= \frac{1}{2z}\left[1 - \frac{1}{z} + \frac{1}{z^2} + \cdots\right] + \frac{1}{2z}\left[1 + \frac{3}{z} + \frac{9}{z^2} + \cdots\right] \quad (3).$$

3A $c(\lambda) = \lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0.$ (1)

given polynomial $p(\lambda)$ (say)

$$p(\lambda) = c(\lambda)\cdot(\lambda^4 + \lambda) + 2\lambda + 1$$

$$\therefore p(A) = \underbrace{c(A)\cdot(A^4 + A)} + 2A + I \quad (3)$$

$$= 0 + 2A + I = \begin{bmatrix} 3 & 6 & 14 \\ 8 & 5 & 6 \\ 2 & 4 & 3 \end{bmatrix} \quad (1)$$

3B Means: $\mu_x = 3$, $\mu_y = 5/3$

$V(X) = \frac{1}{n-1}\sum_{i=1}(x_i - \mu_x)^2 = 1\cancel{2}$; $V(Y) = \cancel{\frac{4}{3}}\ \cancel{3}\ \frac{1}{3}.$

$cov(x,y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y) = \cancel{1}\ \cancel{\frac{3}{2}}\ \frac{1}{2}$

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~~Covariance matrix $\Sigma =$~~

3B means : $\mu_x = 3$, $\mu_y = 5/3$

$V(X) = 1$, $V(Y) = 1/3$, $cov(X, Y) = 1/2$

~~coef~~ covariance matrix $\Sigma = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix}$ 3

eigenvalues of Σ : $\lambda_1 \approx$ ~~12~~ 1.2676

$\lambda_2 \approx 0.0657$

eigenvector of $\lambda_1 \approx \begin{pmatrix} -0.8817 \\ +0.4719 \end{pmatrix}$. — 1

Projected 1D data $= \begin{bmatrix} x_1-\mu_x & y_1-\mu_y \\ x_2-\mu_x & y_2-\mu_y \\ x_3-\mu_x & y_3-\mu_y \end{bmatrix} \begin{bmatrix} -0.8817 \\ -0.4719 \end{bmatrix} = \begin{bmatrix} 1.1962 \\ -0.1573 \\ -1.0390 \end{bmatrix}$ 1.

3C. (i) $|N \cup T \cup F| = 80$ 2

(ii) only one magazine ~~= 56~~.

= only N + only T + only F = 28 + 18 + 10 = 56 3.

3D. $5^{-1} \bmod 17 \equiv 7 \bmod 17$ — 2

$\therefore 5x \equiv 13 \bmod 17 \Rightarrow 35x \equiv$ ~~221~~ $91 \bmod 17$

$\Rightarrow x \equiv 6 \bmod 17$ 3.

$\therefore x = 17 \cdot k + 6 \quad k \in \mathbb{Z}$

3E $v = -e^{-x} \sin y - \frac{1}{2}(x^2 - y^2)$ 5

orthogonal trajectory : $e^{-x} \sin y - \frac{1}{2}(x^2 - y^2) = c_2$

3F $z_1 = -1 - 2i$ lies inside C, $z_2 = -1 + 2i$ lies outside C 1

$Res(f, z_1) = \frac{1+i}{2i}$ 2

$\oint_C f(z) dz = 2\pi i \times \frac{1+i}{2i} = \pi(1+i)$. 2