(Affiliated to University of Mumbai)	***End Semester Examination (R-24) SH 2026 Answer Key with marking scheme***
Branch: EXTC	Course: Electromagnetics Engineering and Antenna
Year/ Semester: TE/V	Course code: ETC503
Time: 03 hours	Marks: 80

		Marks
Q.1	All questions are compulsory. Each sub-question carries 5 marks.	30
A.	Laplace's and Poisson's equations (5 marks) Poisson's equation relates electric potential V to volume charge density ρ_v in a region containing charge: $\nabla^2 V = -\rho_v/\varepsilon$ (for a homogeneous medium). In Cartesian coordinates: $\partial^2V/\partial x^2 + \partial^2V/\partial y^2 + \partial^2V/\partial z^2 = -\rho_v/\varepsilon$. In a charge-free region, $\rho_v = 0$, so Poisson's equation reduces to Laplace's equation: $\nabla^2 V = 0$. Significance: Poisson's equation is used where charge exists; Laplace's equation is used in charge-free regions to determine potential from boundary conditions. Marking: Poisson equation 2; Laplace equation 1; explanation/use 2.	05
B.	Significance of Maxwell's equations for time-varying fields (5 marks) 1. Gauss's electric law: $\nabla \cdot D = \rho_v$; electric charge produces electric flux. 2. Gauss's magnetic law: $\nabla \cdot B = 0$; magnetic field lines form closed loops. 3. Faraday's law: $\nabla \times E = -\partial B/\partial t$; a changing magnetic field induces an electric field. 4. Ampère–Maxwell law: $\nabla \times H = J + \partial D/\partial t$; conduction and displacement currents produce magnetic fields. Together, these laws explain electromagnetic-wave generation and propagation and are fundamental to antennas, transmission lines and wireless communication. Marking: four laws 3; significance/applications 2.	05
C.	Definitions (5 marks) 1. Beamwidth: angular width of the main lobe of an antenna radiation pattern, commonly measured between half-power (−3 dB) points; this is the HPBW. 2. Bandwidth: range of frequencies over which an antenna meets specified performance requirements, such as impedance matching or gain. 3. Aperture efficiency: ratio of effective aperture to physical aperture, $\eta_ap = A_e/A_p$; it indicates how effectively the physical aperture captures or radiates power. Marking: beamwidth 1.5; bandwidth 1.5; aperture efficiency and formula 2.	05
D.	End-fire array and broadside array (5 marks) End-fire array: maximum radiation is directed along the array axis. The elements are usually excited with a suitable progressive phase shift, often approximately $\pm kd$ depending on direction and convention. Broadside array: maximum radiation is perpendicular to the array axis. For a uniform linear array, elements are commonly excited in phase ($\alpha = 0$). Comparison: end-fire maximum is along the axis, while broadside maximum is normal to the axis; their required phase progression and principal radiation directions differ. Both use interference between element fields to shape the radiation pattern.	05

	Marking: end-fire description 1.5; broadside description 1.5; comparison/phase relation 2.	
E.	Gauss's Law – Statement The total electric flux (displacement flux) passing through any closed surface is equal to the total free charge enclosed by that surface. *[1 M]* Derivation Consider a point charge Q at the origin and an imaginary spherical (Gaussian) surface of radius r. From Coulomb's law: $E = Q/(4\pi\varepsilon_0 r^2)\, a_r \;\Rightarrow\; D = \varepsilon_0 E = Q/(4\pi r^2)\, a_r$ On the sphere $dS = r^2 \sin\theta\, d\theta\, d\varphi\, a_r$, hence $\oint D\cdot dS = \int_0^{2\pi}\int_0^{\pi} [Q/(4\pi r^2)]\, r^2 \sin\theta\, d\theta\, d\varphi = [Q/(4\pi)](2\pi)(2) = Q$ For a volume charge distribution $Q_{enc} = \int \rho_v\, dv$, so $\oint D\cdot dS = \int \rho_v\, dv$ (integral form). Applying the divergence theorem $\oint D\cdot dS = \int(\nabla\cdot D)\, dv$, we get $\nabla\cdot D = \rho_v$ (point/differential form). *[2 M]* Calculation of ρ_v $\rho_v = \nabla\cdot D = \partial D_x/\partial x + \partial D_y/\partial y + \partial D_z/\partial z$ $D_x = -5xy\varepsilon_0$, $D_y = -3x^2\varepsilon_0$, $D_z = 8 \;\Rightarrow\; \partial D_x/\partial x = -5y\varepsilon_0$, $\partial D_y/\partial y = 0$, $\partial D_z/\partial z = 0$. So $\rho_v = -5y\varepsilon_0$. *[1 M]* At $(-3, 7, 6)$: $\rho_v = -5(7)\varepsilon_0 = -35\varepsilon_0 = -35 \times 8.854\times10^{-12} = -3.1\times10^{-10}$ C/m^3 (≈ -0.31 nC/m^3). *[1 M]*	05
F.	Ionospheric propagation terms (5 marks) 1. Maximum usable frequency (MUF): the highest frequency that can be used for communication over a specified ionospheric path while still being returned to Earth by refraction. Under the secant-law approximation, MUF = f_c sec i, where f_c is the critical frequency and i is the incidence angle measured from the normal. 2. Virtual height: the apparent height from which a returned radio wave would seem to be reflected if it travelled in a straight line at the speed of light. It is inferred from the measured time delay and is generally different from the actual height of the ionospheric layer. Marking: MUF definition/formula 2.5; virtual height definition 2.5.	05
Q.2A	Electromagnetic boundary conditions at a dielectric–conductor interface (10 marks) Let n̂ point from medium 1 to medium 2. General boundary conditions are: 1. Tangential electric field: $\hat{n}\times(E_2 - E_1) = 0$, from Faraday's law for an infinitesimal loop. Thus $E_1t = E_2t$ for ordinary finite fields. 2. Normal electric flux density: $\hat{n}\cdot(D_2 - D_1) = \rho_s$, from Gauss's law, where ρ_s is free surface-charge density. 3. Tangential magnetic field: $\hat{n}\times(H_2 - H_1) = J_s$, where J_s is surface current density, from Ampère's law. 4. Normal magnetic flux density: $\hat{n}\cdot(B_2 - B_1) = 0$, because magnetic monopoles do not exist; hence $B_1n = B_2n$. At an ideal perfect conductor, E = 0 inside the conductor. Therefore tangential E at its surface is zero; the normal D just outside is related to surface charge. The tangential H discontinuity is determined by surface current, while normal B is continuous. For a real conductor, fields penetrate to a finite skin depth, so the ideal-conductor approximation applies when appropriate.	10

	Marking: tangential E condition 2; normal D condition 2; tangential H condition 2; normal B condition 2; interpretation at conductor 2.	
	OR	
Q.2B.	Poynting theorem and electromagnetic energy flow (10 marks) Poynting theorem expresses conservation of electromagnetic energy. In differential form: $\nabla\cdot(E \times H) = -E\cdot J - \partial/\partial t\,[\tfrac{1}{2}(E\cdot D + B\cdot H)]$ (for a linear, nondispersive medium; the stored-energy term is written in this standard form). The Poynting vector is $S = E \times H$ and its unit is W/m^2. Its direction indicates the direction of electromagnetic power flow and its magnitude is power crossing unit area. Integral form: $\oint_S S\cdot dA = -\int_V E\cdot J\ dV - d/dt \int_V u_{em}\ dV,$ where u_{em} is stored electromagnetic energy density. In words, net power flowing out of a closed surface equals the decrease in stored field energy minus power dissipated in the medium (equivalently, input power is divided between radiation, losses and change in stored energy). For a transmitting antenna, current produces E and H fields; their cross product points outward in the radiating region, carrying energy into free space. Some supplied power is dissipated as conductor/dielectric heat, while the remaining power is radiated. In steady-state time-average operation, the net outward Poynting flux represents radiated power, with losses accounted for separately. Marking: theorem statement/meaning 2; differential form 2; Poynting vector 2; integral/energy balance 2; antenna application 2.	10
Q.3A	Radiation field of a dipole antenna (10 marks) Consider a short z-directed dipole of length l carrying current I_0, with $l \ll \lambda$. In the far field, only the transverse components $E\theta$ and $H\varphi$ are significant; radial field components are negligible compared with them. For a Hertzian dipole, the far-zone electric field is: $E\theta = j\eta_0\, k\, I_0\, l \sin\theta\ e^{(-jkr)}/(4\pi r),$ with the sign/phase depending on the adopted time convention. Here $k = 2\pi/\lambda$, $\eta_0 = \sqrt{(\mu_0/\varepsilon_0)} \approx 120\pi\ \Omega$, and r is the distance from the dipole. The magnetic field is related by $H\varphi = E\theta/\eta_0$. Thus: $H\varphi = j\, k\, I_0\, l \sin\theta\ e^{(-jkr)}/(4\pi r).$ The radiation pattern magnitude is proportional to $\sin\theta$: it is maximum at $\theta = 90°$ (broadside) and zero along the dipole axis at $\theta = 0°$ and 180°. The radiated fields vary as 1/r and E and H are mutually perpendicular and perpendicular to the propagation direction. Marking: assumptions/coordinates 1; far-field $E\theta$ derivation/result 4; $H\varphi$ relation 2; pattern/null/max interpretation 2; definitions/field behavior 1.	10
	OR	
Q.3B	Radiation resistance of an infinitesimal dipole (10 marks) For a Hertzian dipole of length $l \ll \lambda$ with uniform current I_0, the far-field magnitude is $\lvert E\theta\rvert = \eta_0 k I_0 l \sin\theta/(4\pi r)$, where $k = 2\pi/\lambda$. Time-average power density for peak phasors is $S_{av} = \lvert E\theta\rvert^2/(2\eta_0)$. Total radiated power is $P_{rad} = \int_0^{2\pi}\int_0^{\pi} S_{av}\, r^2 \sin\theta\ d\theta\ d\varphi.$	10

	Using $\int_0^{2\pi} d\varphi = 2\pi$ and $\int_0^{\pi} \sin^3\theta \, d\theta = 4/3$ gives $P_{rad} = \eta_0 k^2 I_0^2 l^2/(12\pi) = 40\pi^2 I_0^2 (l/\lambda)^2$. Radiation resistance is defined by $P_{rad} = \frac{1}{2} I_0^2 R_r$ for peak current. Therefore, $R_r = 2P_{rad}/I_0^2 = 80\pi^2 (l/\lambda)^2 \ \Omega$. This $80\pi^2$ result applies to a Hertzian dipole with uniform current. For a short dipole with triangular current distribution, the commonly used result is approximately $R_r = 20\pi^2 (l/\lambda)^2 \ \Omega$. Marking: far-field expression 2; power density/integral 2; angular integration 2; radiated power 2; radiation resistance and distinction 2.	
Q.4A	Four-element isotropic linear array: $d = \lambda/2$, $\alpha = 0$ (10 marks) For N = 4 equally spaced isotropic sources along the z-axis, the array factor is $AF = \sin(N\psi/2)/\sin(\psi/2)$, where $\psi = kd\cos\theta + \alpha$. Given $d = \lambda/2$, $kd = \pi$ and $\alpha = 0$, so $\psi = \pi\cos\theta$ and $AF = \sin(2\pi\cos\theta)/\sin(\pi\cos\theta/2)$. Nulls occur when $\sin(N\psi/2) = 0$, excluding directions where numerator and denominator vanish together. Thus $\psi = 2m\pi/N = m\pi/2$ for $m = \pm1, \pm2, \ldots$, excluding the principal maximum condition. With $\psi = \pi\cos\theta$, $\cos\theta = m/2$. Valid null directions in 0°–180° are $\theta = 60°, 90°, 120°$. Principal maxima occur at $\psi = 0$ (broadside), giving $\theta = 90°$. The end-fire directions $\theta = 0°, 180°$ are also evaluated by the array-factor limit and are not principal maxima for this four-element in-phase array. For a uniform broadside array, approximate HPBW formula is HPBW $\approx 2\sin^{-1}[0.443\lambda/(Nd)]$ radians. With N = 4 and $d = \lambda/2$, HPBW $\approx 2\sin^{-1}(0.2215) \approx 25.6°$. FNBW is the angular separation between the first nulls on either side of the main beam: $120° - 60° = 60°$. Marking: array-factor expression 2; nulls 3; maxima 2; HPBW calculation 2; FNBW 1.	10
	OR	
Q.4B	Two equal-amplitude, in-phase isotropic sources (10 marks) Consider two isotropic point sources separated by distance d along an array axis and excited with equal amplitudes and the same phase ($\alpha = 0$). Let θ be measured from the array axis and $k = 2\pi/\lambda$. The total far electric field is the phasor sum: $E = (E_0 e^{-jkr}/r)[e^{(jkd\cos\theta/2)} + e^{(-jkd\cos\theta/2)}]$. Using $e^{jx} + e^{-jx} = 2\cos x$, $E = (2E_0 e^{-jkr}/r)\cos[(kd/2)\cos\theta]$. Hence the array factor is $AF = 2\cos[(kd/2)\cos\theta]$, and its normalized magnitude is $\lvert\cos[(kd/2)\cos\theta]\rvert$. For $d = \lambda/2$, $kd = \pi$, so $AF = 2\cos[(\pi/2)\cos\theta]$. The maximum occurs broadside at $\theta = 90°$, where the path difference is zero. Nulls satisfy $(kd/2)\cos\theta = (2m+1)\pi/2$; for $d = \lambda/2$, the end-fire directions $\theta = 0°$ and 180° are nulls. Marking: setup/phase 2; field sum 3; array factor 2; normalized pattern 1; maxima/nulls 2.	10
Q.5A	Circular microstrip patch antenna design and feeding techniques (10 marks) The question does not specify operating frequency or substrate, so a unique numerical radius cannot be determined. Award full design-method credit when suitable assumptions are stated. For illustration, assume $f_r = 2.4$ GHz, $\varepsilon_r = 4.4$ (FR-4), and h =	10

	1.6 mm; other consistent assumptions are acceptable. For the dominant TM_{11} mode, define $F = 8.791\times10^9/(f_r\sqrt{\varepsilon_r})$ cm, $a = F / \{1 + [2h/(\pi\varepsilon_r F)]\ [\ln(\pi F/(2h)) + 1.772]\}^{(1/2)}$, where a is the physical patch radius (use consistent units). Calculate F and then a using the selected parameters; the radius is a starting design value and should be tuned in simulation to account for fringing and material tolerances. Feeding techniques: 1. Coaxial probe feed: probe connects patch to coax centre conductor; feed position sets input impedance; low spurious radiation but probe inductance may affect thick substrates. 2. Microstrip-line feed: planar strip connects to patch; simple fabrication/integration but may add feed radiation and has limited bandwidth. 3. Aperture-coupled feed: microstrip feed couples through a slot in the ground; offers feed/patch isolation but requires multilayer construction. 4. Proximity-coupled feed: feed line is beneath patch and couples electromagnetically; can provide wider bandwidth but alignment is more demanding. 5. Inset feed: a microstrip line enters the patch by a chosen depth to achieve impedance matching. Marking: design assumptions/mode 1; formula and calculation approach 3; four feeding methods and explanation 5; tuning/ground-plane note 1.	
	OR	
Q.5B	Rectangular microstrip patch antenna design at 2.4 GHz (10 marks) Given: $f_r = 2.4$ GHz, $\varepsilon_r = 4.4$, $h = 0.16$ cm $= 1.6$ mm, $c = 3\times10^8$ m/s. 1. Width: $W = c/(2f_r)\ \sqrt{[2/(\varepsilon_r+1)]}$ $= (3\times10^8)/(2\times2.4\times10^9)\ \sqrt{(2/5.4)} \approx 38.0$ mm. 2. Effective dielectric constant: $\varepsilon_eff = (\varepsilon_r+1)/2 + (\varepsilon_r-1)/2\ [1 + 12h/W]^{(-1/2)}$ $\approx 4.4/2$? Substitution gives $\varepsilon_eff \approx 4.08$. 3. Fringing extension: $\Delta L/h = 0.412\ [(\varepsilon_eff+0.3)(W/h+0.264)]/[(\varepsilon_eff-0.258)(W/h+0.8)]$. With $W/h \approx 23.75$, $\Delta L/h \approx 0.46$, so $\Delta L \approx 0.74$ mm. 4. Effective length: $L_eff = c/(2f_r\sqrt{\varepsilon_eff}) \approx 31.0$ mm. 5. Actual length: $L = L_eff - 2\Delta L \approx 31.0 - 1.48 = 29.5$ mm. Therefore, initial patch dimensions are $W \approx 38.0$ mm and $L \approx 29.5$ mm. Use a ground plane larger than the patch and optimize the feed location for 50-Ω matching. Marking: width formula/calculation 2; ε_eff 2; ΔL 2; L_eff 2; actual length and design note 2.	10
Q.6A	Sky-wave propagation through D, E and F regions (10 marks) Sky-wave propagation uses refraction in the ionosphere to return HF radio waves to Earth, enabling communication beyond the horizon. A ray enters the ionosphere and is progressively refracted by the electron-density gradient; it bends back toward Earth rather than reflecting from a sharp physical surface. D region (about 60–90 km): present mainly during daytime; causes significant absorption of lower-frequency HF waves and largely disappears at night.	10

	E region (about 90–140 km): can refract/return some radio waves; its influence is generally stronger in daytime and sporadic-E layers may occur. F region (roughly 140–400 km or higher): most important for long-distance HF communication. By day it may split into F1 and F2 layers; at night these often merge into one F layer. Its higher electron density supports higher critical frequencies and longer hops. Multiple-hop propagation occurs when a sky wave returns to Earth and is reflected again by the ground, then refracted by the ionosphere repeatedly. This permits communication over very long distances; each hop depends on frequency, ionospheric conditions, ground reflection and losses. A ray diagram should show ground stations, the D/E/F layers, upward and returning ray paths, and one or more ground reflections. Marking: principle 2; D region 1.5; E region 1.5; F region 2; multiple reflections/ray diagram 3.	
	OR	
Q.6B	a) Duct propagation (5 marks) Duct propagation occurs when atmospheric refractive-index variations create a layer or "duct" that traps or guides radio waves, causing them to travel over distances beyond the normal radio horizon. It can occur due to temperature inversions and humidity gradients, especially over sea surfaces. Waves are refracted repeatedly within the duct, reducing energy escape and enabling extended coverage. It is frequency- and weather-dependent and may cause unexpected interference or coverage gaps. Marking: definition 1; mechanism/conditions 2; applications/effects/limitations 2. b) Friis transmission formula (5 marks) For free-space line-of-sight propagation with matched, aligned antennas and far-field separation R, the received power is $P_r = P_t G_t G_r (\lambda/(4\pi R))^2$, where P_t is transmitted power, G_t and G_r are transmitter and receiver gains, λ is wavelength, and R is separation. Equivalently, free-space path loss is $(4\pi R/\lambda)^2$ and $P_r = P_t G_t G_r / L_fs$. Derivation: power density at distance R from an isotropic transmitter is $P_t/(4\pi R^2)$. A transmitting antenna of gain G_t gives $S = P_t G_t/(4\pi R^2)$. The receiving antenna effective aperture is $A_e = G_r\lambda^2/(4\pi)$. Hence $P_r = SA_e = P_t G_t G_r \lambda^2/(4\pi R)^2$. Assumptions: free space, far field, polarization matched, impedance matched, and no additional losses. Marking: formula/definitions 2; power-density derivation 2; assumptions 1.	10
	*********** All the Best***********	